

FRM Part 1 Formula Sheet 2026

More than 370 formulas for FRM Part 1, set out chapter by chapter across Foundations of Risk Management, Quantitative Analysis, Financial Markets and Products, and Valuation and Risk Models. Every formula is taken from the 2026 GARP readings in their own notation, and the full sheet can be read right here or downloaded as one PDF.

4 books **62** chapters reviewed **370+** formulas **Oct 2026** last updated

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BOOK 1 OF 4 • 20% OF THE EXAM

Foundations of Risk Management formulas

FRM 1 The Building Blocks of Risk Management

1 formula

RAROC

$$\text{RAROC} = \frac{\text{Reward}}{\text{Risk}}$$

in full: $\frac{\text{After-tax net risk-adjusted expected return}}{\text{Economic capital}}$

FRM 2 How Do Firms Manage Financial Risk?

Concepts only

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

FRM 3 The Governance of Risk Management

Concepts only

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

FRM 4 Credit Risk Transfer Mechanisms

Concepts only

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

FRM 5 Modern Portfolio Theory and the CAPM

10 formulas

Beta

$$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2} = \rho_{i,M} \frac{\sigma_i}{\sigma_M}, \beta_M = 1$$

Portfolio beta

$$\beta_P = \sum w_i \beta_i$$

CAPM (SML)

$$E(R_i) = r_f + \beta_i [E(R_M) - r_f]$$

$$\text{slope of the SML} = E(R_M) - r_f$$

CML

$$E(R_P) = r_f + \frac{E(R_M) - r_f}{\sigma_M} \sigma_P$$

$$\text{slope of the CML} = [E(R_M) - r_f] / \sigma_M$$

Sharpe performance index

$$\text{SPI} = \frac{E(R_P) - r_f}{\sigma_P}$$

Treynor performance index

$$\text{TPI} = \frac{E(R_P) - r_f}{\beta_P}$$

Jensen performance index

$$R_P - r_f = \alpha_P + \beta_P(R_M - r_f)$$

$$\alpha_P = E(R_P) - \{r_f + \beta_P[E(R_M) - r_f]\}$$

$\alpha_P = 0$ in equilibrium; significantly positive is superior, negative is inferior

Tracking error

$$\text{TE} = \sigma(R_P - R_B)$$

Information ratio

$$\text{IR} = \frac{E(R_P - R_B)}{\sigma(R_P - R_B)}$$

active return over tracking error

Sortino ratio

$$\text{SR} = \frac{E(R_P) - \text{MAR}}{\sqrt{\text{MSD}_{\min}}} \quad \text{MSD}_{\min} = \frac{1}{N} \sum_{t=1}^N \min(0, R_{Pt} - \text{MAR})^2$$

EXAM TIP Sharpe divides by total risk σ_P ; Treynor divides by systematic risk β_P , so it suits a well-diversified portfolio.

FRM 6

The APT and Multifactor Models

4 formulas

APT return model

$$R_i = E(R_i) + \beta_{i1}[I_1 - E(I_1)] + \dots + \beta_{iK}[I_K - E(I_K)] + e_i$$

APT expected return

$$E(R_P) = E(R_Z) + \beta_{P1}[E(I_1) - E(R_Z)] + \dots + \beta_{PK}[E(I_K) - E(R_Z)]$$

$E(R_Z)$ = zero-beta (risk-free) return; single factor: $K = 1$

Fama-French three-factor

$$E(R_P) = r_f + \alpha_P + \beta_{P,M}[E(R_M) - r_f] + \beta_{P,SMB} E(\text{SMB}) + \beta_{P,HML} E(\text{HML})$$

Portfolio factor beta

$$\beta_{P,k} = \sum w_i \beta_{i,k}$$

a hedge adds positions until $\beta_{P,k} = 0$ for each factor hedged

FRM 7

Principles for Effective Data Aggregation and Risk Reporting

Concepts only

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

FRM 8

Enterprise Risk Management and Future Trends

Concepts only

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

FRM 9

Learning from Financial Disasters

Concepts only

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

FRM 10

Anatomy of the Great Financial Crisis of 2007-2009

Concepts only

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

FRM 11

GARP Code of Conduct

Concepts only

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

Quantitative Analysis formulas

QTA 1 Fundamentals of Probability

8 formulas

Complement	$P(A^c) = 1 - P(A)$
Addition rule	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Conditional probability	$P(A B) = \frac{P(A \cap B)}{P(B)}$
Total probability	$P(B) = P(B A_1)P(A_1) + \dots + P(B A_n)P(A_n)$ A_i mutually exclusive and exhaustive
Total probability, two events	$P(A) = P(A B)P(B) + P(A B^c)P(B^c)$
Independence	$P(A \cap B) = P(A)P(B)$, i.e. $P(A B) = P(A)$ n events: $P(A_1 \cap \dots \cap A_n) = P(A_1) \dots P(A_n)$
Conditional independence	$P(A \cap B C) = P(A C)P(B C)$
Bayes' rule	$P(B A) = \frac{P(A B)P(B)}{P(A)}$

EXAM TIP Mutually exclusive is not independent: two mutually exclusive events with positive probabilities are dependent.

QTA 2 Random Variables

10 formulas

Bernoulli PMF	$f(x) = p^x(1-p)^{1-x}, x \in \{0, 1\}$
Expectation	$E[X] = \sum x_i p_i \quad E[f(X)] = \sum f(x_i) p_i$
Properties of E	$E[a + bX + cY] = a + bE[X] + cE[Y]$ $E[g(X)] \neq g(E[X])$ in general
Variance	$\sigma^2 = E[(X - \mu)^2] = E[X^2] - (E[X])^2$
Skewness	$\text{Skew}(X) = \frac{E[(X - \mu)^3]}{\sigma^3}$ numerator is the third central moment
Kurtosis	$\text{Kurt}(X) = \frac{E[(X - \mu)^4]}{\sigma^4} \quad \text{excess kurtosis} = \text{Kurt} - 3$
Standardized variable	$Z = \frac{X - \mu}{\sigma}$ mean 0, variance 1

Linear transformation	$Y = a + bX: E[Y] = a + bE[X], V[Y] = b^2V[X], \sigma_Y = b \sigma_X$
Higher moments of $Y = a + bX$	skewness unchanged if $b > 0$ (sign flips if $b < 0$) kurtosis unchanged median and IQR move like the mean and SD
Median and IQR	median = 50% quantile IQR = $q_{0.75} - q_{0.25}$

QTA 3

Common Univariate Random Variables

23 formulas

Binomial	$P(X = x) = {}^nC_x p^x (1-p)^{n-x}$
Poisson	$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$ over a period t , use λt
Uniform, interval probability	$P(l < X < u) = \frac{\min(u, b) - \max(l, a)}{b - a}$
Normal	$P(X \leq x) = \Phi((x - \mu)/\sigma) \quad \phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$
Normal facts	mean = median = mode skewness 0 kurtosis 3 $\Phi(-z) = 1 - \Phi(z)$
Normal probability ranges	68%: $\mu \pm \sigma$ 90%: $\mu \pm 1.645\sigma$ 95%: $\mu \pm 1.96\sigma$ 99%: $\mu \pm 2.58\sigma$
Lognormal	$P(X \leq x) = \Phi((\ln x - \mu)/\sigma)$ positively skewed
Exponential	$P(X \leq x) = 1 - e^{-x/\beta}, x \geq 0$
Student's t kurtosis	$\text{Kurt} = \frac{3(\nu - 2)}{\nu - 4}$ for $\nu > 4$, always above 3
Constructions	$\chi_\nu^2 = \sum_{i=1}^\nu Z_i^2 \quad t_\nu = \frac{Z}{\sqrt{W/\nu}}, W \sim \chi_\nu^2 \quad F_{\nu_1, \nu_2} = \frac{W_1/\nu_1}{W_2/\nu_2}$
Mixture	$f(x) = \sum w_i f_i(x) \quad E[X] = \sum w_i \mu_i$ $V[X] = \sum w_i (\sigma_i^2 + \mu_i^2) - (\sum w_i \mu_i)^2$

Distribution	Mean	Variance
Bernoulli	p	$p(1 - p)$
Binomial	np	$np(1 - p)$
Poisson	λ	λ
Uniform $[a, b]$	$(a + b)/2$	$(b - a)^2/12$
Normal	μ	σ^2
Standard normal	0	1
Lognormal	$e^{\mu + \sigma^2/2}$	$(e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$
Student's t_ν	0	$\nu/(\nu - 2)$
F_{ν_1, ν_2}	$\nu_2/(\nu_2 - 2)$	not required
χ_ν^2	ν	2ν
Exponential	β	β^2
Beta(α, β)	$\alpha/(\alpha + \beta)$	not required

EXAM TIP In the lognormal, μ and σ are the mean and SD of $\ln X$, not of X .

QTA 4

Multivariate Random Variables

10 formulas

Marginal and conditional PMF	$f_{X_1}(x_1) = \sum_{x_2} f(x_1, x_2) \quad f(x_1 X_2 = x_2) = \frac{f(x_1, x_2)}{f_{X_2}(x_2)}$
Conditional expectation	$E[X_1 X_2 = x_2] = \sum x_1 f(x_1 X_2 = x_2)$
Conditional variance	$V[X_1 X_2 = x_2] = E[X_1^2 X_2 = x_2] - (E[X_1 X_2 = x_2])^2$
Covariance	$\text{Cov}(X_1, X_2) = E[(X_1 - E[X_1])(X_2 - E[X_2])] = E[X_1 X_2] - E[X_1]E[X_2]$
Correlation	$\text{Corr}(X_1, X_2) = \frac{\text{Cov}(X_1, X_2)}{\sigma_1 \sigma_2}$ so $\text{Cov} = \text{Corr} \cdot \sigma_1 \sigma_2$
Linear transformations	$\text{Cov}(a + bX_1, c + dX_2) = bd \text{Cov}(X_1, X_2)$ $\text{Corr}(aX_1, bX_2) = \frac{ab}{ a b } \text{Corr}(X_1, X_2)$ unchanged if a, b have the same sign, flips sign otherwise
Covariance with a sum	$\text{Cov}(X_1, X_2 + X_3) = \text{Cov}(X_1, X_2) + \text{Cov}(X_1, X_3)$
Variance of a sum or difference	$V[X_1 \pm X_2] = V[X_1] + V[X_2] \pm 2 \text{Cov}(X_1, X_2)$
Variance of a weighted sum	$V[aX_1 + bX_2] = a^2 V[X_1] + b^2 V[X_2] + 2ab \text{Cov}(X_1, X_2)$
Sum of n iid variables	$E[\sum X_i] = n\mu \quad V[\sum X_i] = n\sigma^2$

Sample mean	$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n X_i$
Bias	$\text{Bias}(\hat{\theta}) = E[\hat{\theta}] - \theta \quad \text{Bias}(\hat{\mu}) = 0 \quad \text{Bias}(\hat{\sigma}^2) = -\sigma^2/n$
Variance estimators	$\hat{\sigma}^2 = \frac{1}{n} \sum (X_i - \hat{\mu})^2$ (biased) $s^2 = \frac{1}{n-1} \sum (X_i - \hat{\mu})^2$ (unbiased)
Standard error of the mean	$\sigma/\sqrt{n}$
Central limit theorem	$\hat{\mu} \approx N(\mu, \sigma^2/n)$ for large n $\sqrt{n}(\hat{\mu} - \mu)/\sigma \rightarrow N(0, 1)$
Scaling the mean	$\hat{\mu}_{\text{annual}} = 252 \hat{\mu}_{\text{daily}} = 52 \hat{\mu}_{\text{weekly}}$ n periods: $n\mu$
Scaling volatility	$\sigma_{\text{annual}} = \sqrt{252} \sigma_{\text{daily}} = \sqrt{52} \sigma_{\text{weekly}} = \sqrt{12} \sigma_{\text{monthly}}$ n periods: $\sqrt{n} \sigma$

EXAM TIP Square-root-of-time scaling assumes iid returns; it fails with autocorrelation.

Sample skewness and kurtosis	$\hat{S} = \frac{\frac{1}{n} \sum (X_i - \hat{\mu})^3}{\hat{\sigma}^3} \quad \hat{\kappa} = \frac{\frac{1}{n} \sum (X_i - \hat{\mu})^4}{\hat{\sigma}^4}$
Median	n odd: $x_{((n+1)/2)}$ n even: $\frac{x_{(n/2)} + x_{(n/2+1)}}{2}$
Sample covariance	$\hat{\sigma}_{XY} = \frac{1}{n-1} \sum_{i=1}^n (X_i - \hat{\mu}_X)(Y_i - \hat{\mu}_Y)$
Sample correlation	$\hat{\rho}_{XY} = \frac{\hat{\sigma}_{XY}}{\hat{\sigma}_X \hat{\sigma}_Y}$

Hypotheses	two-sided $H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$ one-sided $H_0 : \mu \leq \mu_0$ vs $H_1 : \mu > \mu_0$ (or the reverse)
Test statistic, mean	$T = \frac{\hat{\mu} - \mu_0}{\sqrt{\hat{\sigma}^2/n}} \sim N(0, 1)$ asymptotically with s^2 from a normal sample, $T \sim t_{n-1}$

Population	σ^2	$n \geq 30$	$n < 30$
Normal	Known	z	z
Non-normal	Known	z	NA
Normal	Unknown	t (or z)	t
Non-normal	Unknown	t (or z)	NA

Critical z	10%	5%	1%
One-sided	1.28	1.645	2.33
Two-sided	1.645	1.96	2.58

Decision	H_0 true	H_0 false
Do not reject H_0	Correct, $1 - \alpha$	Type II error, β
Reject H_0	Type I error, α (size)	Correct, power $1 - \beta$

Confidence interval, two-sided $\hat{\mu} \pm C_\alpha \hat{\sigma} / \sqrt{n}$

Confidence interval, one-sided $H_1 : \mu > \mu_0: [\hat{\mu} - C_\alpha \hat{\sigma} / \sqrt{n}, \infty)$ $H_1 : \mu < \mu_0: (-\infty, \hat{\mu} + C_\alpha \hat{\sigma} / \sqrt{n}]$

p-value two-sided $2[1 - \Phi(|T|)]$ lower tail $\Phi(T)$ upper tail $1 - \Phi(T)$

Difference in means, paired $T = \frac{\hat{\mu}_X - \hat{\mu}_Y}{\sqrt{(\hat{\sigma}_X^2 + \hat{\sigma}_Y^2 - 2\hat{\sigma}_{XY})/n}}$

Difference in means, independent $T = \frac{\hat{\mu}_X - \hat{\mu}_Y}{\sqrt{\hat{\sigma}_X^2/n_X + \hat{\sigma}_Y^2/n_Y}}$

QTA 7

Linear Regression

7 formulas

OLS estimates $\hat{\beta} = \frac{\sum(x_i - \bar{X})(y_i - \bar{Y})}{\sum(x_i - \bar{X})^2} = \frac{\hat{\sigma}_{XY}}{\hat{\sigma}_X^2} = \hat{\rho}_{XY} \frac{\hat{\sigma}_Y}{\hat{\sigma}_X}$ $\hat{\alpha} = \bar{Y} - \hat{\beta} \bar{X}$

Variance of the shocks $s^2 = \frac{1}{n-2} \sum \hat{\varepsilon}_i^2 = \frac{n}{n-2} \hat{\sigma}_Y^2 (1 - R^2)$

Standard error of the slope $\text{s.e.}(\hat{\beta}) = \frac{s}{\sqrt{n} \hat{\sigma}_X}$

t test, one coefficient $T = \frac{\hat{\beta} - \beta_0}{\text{s.e.}(\hat{\beta})}$

same form for $\hat{\alpha}$

Confidence interval, coefficient $\hat{\beta} \pm C_\alpha \text{s.e.}(\hat{\beta})$
 C_α = two-sided critical value (1.96 at 95%)

Sums of squares $\text{ESS} = \sum(\hat{Y}_i - \bar{Y})^2$ $\text{RSS} = \sum(Y_i - \hat{Y}_i)^2$
 $\text{TSS} = \sum(Y_i - \bar{Y})^2 = \text{ESS} + \text{RSS}$

R^2 $R^2 = \frac{\text{ESS}}{\text{TSS}} = 1 - \frac{\text{RSS}}{\text{TSS}}$ one regressor: $R^2 = \hat{\rho}_{XY}^2$

QTA 8

Regression with Multiple Explanatory Variables

5 formulas

R^2 , multiple regressors $R^2 = \hat{\rho}^2(Y, \hat{Y})$

Adjusted R^2 $\bar{R}^2 = 1 - \frac{\text{RSS}/(n-k-1)}{\text{TSS}/(n-1)} = 1 - \frac{n-1}{n-k-1} (1 - R^2)$

k excludes the constant

F test, q restrictions

$$F = \frac{(\text{RSS}_R - \text{RSS}_U)/q}{\text{RSS}_U/(n - k_U - 1)} = \frac{(R_U^2 - R_R^2)/q}{(1 - R_U^2)/(n - k_U - 1)}$$

$$F \sim F_{q, n-k_U-1}$$

F statistic, all slopes zero

$$F = \frac{(\text{TSS} - \text{RSS})/k}{\text{RSS}/(n - k - 1)} \sim F_{k, n-k-1}$$

F from t, uncorrelated regressors

$$F \approx \frac{T_1^2 + T_2^2}{2}$$

QTA 9

Regression Diagnostics

4 formulas

Omitted variable bias

$$\hat{\beta}_1 \rightarrow \beta_1 + \beta_2 \delta, \delta = \frac{\text{Cov}(X_1, X_2)}{V(X_1)}$$

White test

nR^2 from regressing $\hat{\varepsilon}^2$ on the regressors, their squares and cross products $\sim \chi_{k(k+3)/2}^2$

Variance inflation factor

$$\text{VIF}_j = \frac{1}{1 - R_j^2}$$

above 10 is excessive

Cook's distance

$$D_j = \frac{\sum_{i=1}^n (\hat{Y}_i^{(-j)} - \hat{Y}_i)^2}{k s^2}$$

$D_j > 1$ flags an outlier

QTA 10

Stationary Time Series

19 formulas

Autocorrelation

$$\rho_h = \gamma_h / \gamma_0$$

AR(1)

$$Y_t = \delta + \phi Y_{t-1} + \varepsilon_t, \varepsilon_t \sim WN(0, \sigma^2)$$

$|\phi| < 1$ stationary, $|\phi| = 1$ non-stationary

AR(1) moments

$$\mu = \frac{\delta}{1 - \phi} \quad \gamma_0 = \frac{\sigma^2}{1 - \phi^2} \quad \gamma(h) = \phi^{|h|} \gamma_0 \quad \rho(h) = \phi^{|h|}$$

μ is the mean-reverting level

AR(1) PACF

$$\alpha(h) = \phi^{|h|} \text{ for } |h| \leq 1 \quad 0 \text{ for } |h| \geq 2$$

AR(p)

$$Y_t = \delta + \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + \varepsilon_t$$

AR(p) moments

$$\mu = \frac{\delta}{1 - \phi_1 - \dots - \phi_p} \quad \gamma_0 = \frac{\sigma^2}{1 - \phi_1 \rho_1 - \dots - \phi_p \rho_p}$$

AR(p) ACF and PACF

ACF decays, non-zero at all lags PACF $\alpha(h) = 0$ for $h > p$

Characteristic equation

$$\text{roots of } z^p - \phi_1 z^{p-1} - \dots - \phi_p = 0$$

stationary if every root has $|z| < 1$

$$\text{AR}(2): z^2 - \phi_1 z - \phi_2 = 0$$

MA(1)	$Y_t = \mu + \theta \varepsilon_{t-1} + \varepsilon_t$, always covariance stationary $\theta > 0$ persistent, $\theta < 0$ mean-reverting
MA(1) moments	$E[Y_t] = \mu \quad \gamma_0 = (1 + \theta^2)\sigma^2 \quad \rho(1) = \frac{\theta}{1 + \theta^2}, \rho(h) = 0 \text{ for } h \geq 2$
MA(q)	$Y_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q} \quad E[Y_t] = \mu$ $\gamma_0 = \sigma^2(1 + \theta_1^2 + \dots + \theta_q^2)$
MA(q) ACF and PACF	$\rho(h) = 0 \text{ for } h > q \quad \text{PACF decays, non-zero at all lags}$
ARMA(1,1)	$Y_t = \delta + \phi Y_{t-1} + \theta \varepsilon_{t-1} + \varepsilon_t \quad \mu = \frac{\delta}{1 - \phi} \quad \gamma_0 = \frac{\sigma^2(1 + 2\phi\theta + \theta^2)}{1 - \phi^2}$ $\gamma(h) = \phi \gamma(h-1) \text{ for } h \geq 2$
EXAM TIP Identification: the PACF of an AR(p) cuts off after lag p ; the ACF of an MA(q) cuts off after lag q .	
Lag operator	$LY_t = Y_{t-1} \quad 1 - a_1 L \text{ is invertible if } a_1 < 1$
Sample autocorrelation	$\hat{\rho}_h = \hat{\gamma}_h / \hat{\gamma}_0$
Box-Pierce	$Q_{BP} = T \sum_{i=1}^h \hat{\rho}_i^2 \sim \chi_h^2$
Ljung-Box	$Q_{LB} = T(T+2) \sum_{i=1}^h \frac{\hat{\rho}_i^2}{T-i} \sim \chi_h^2$ $Q_{BP} \approx Q_{LB} \text{ in large samples}$
Model selection	$\hat{\sigma}^2 = \frac{1}{T} \sum_{t=1}^T \hat{\varepsilon}_t^2 \quad \text{AIC} = T \ln \hat{\sigma}^2 + 2k \quad \text{BIC} = T \ln \hat{\sigma}^2 + k \ln T$
Forecasts	AR(1): $E_T[Y_{T+h}] = \mu + \phi^h(Y_T - \mu) \rightarrow \mu$ ARMA(1,1), one step: $E_T[Y_{T+1}] = \delta + \phi Y_T + \theta \varepsilon_T$

QTA 11	Non-Stationary Time Series	7 formulas
Linear trend	$Y_t = \delta_0 + \delta_1 t + \varepsilon_t \quad E[Y_t] = \delta_0 + \delta_1 t$	
Exponential and log-linear trend	$Y_t = \beta_0 e^{\beta_1 t} \Leftrightarrow \ln Y_t = \ln \beta_0 + \beta_1 t$	
Seasonal dummies	$Y_t = \delta + \gamma_1 I_{1t} + \dots + \gamma_{s-1} I_{s-1,t} + \varepsilon_t$	
Random walk	$Y_t = Y_0 + \sum_{i=1}^t \varepsilon_i \quad V[Y_t] = t\sigma^2$ $\Delta Y_t = Y_t - Y_{t-1}$ removes a unit root	
ADF test	$\Delta Y_t = \gamma Y_{t-1} + (\delta_0 + \delta_1 t) + \lambda_1 \Delta Y_{t-1} + \dots + \lambda_p \Delta Y_{t-p} + \varepsilon_t$ statistic: t of $\hat{\gamma}$; $H_0 : \gamma = 0$ (unit root), $H_1 : \gamma < 0$ (stationary)	
Forecasts	trend: $E_T[Y_{T+h}] = \delta_0 + \delta_1(T+h)$ seasonal: $E_T[Y_{T+h}] = \delta + \gamma_j, j = (T+h) \bmod s$	

95% forecast interval

$$E_T[Y_{T+h}] \pm 1.96 \sigma$$

QTA 12

Measuring Returns, Volatility and Correlation

6 formulas

Simple return

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad 1 + R_T = \prod_{t=1}^T (1 + R_t)$$

Log return

$$r_t = \ln P_t - \ln P_{t-1} \quad r_T = \sum_{t=1}^T r_t \quad 1 + R_t = e^{r_t}$$

Jarque-Bera

$$JB = (T-1) \left[\frac{\hat{S}^2}{6} + \frac{(\hat{\kappa} - 3)^2}{24} \right] \sim \chi_2^2$$

5% critical value 5.99

Power law

$$P(X > x) = kx^{-\alpha} \Leftrightarrow \ln P(X > x) = \ln k - \alpha \ln x$$

Spearman rank correlation

$$\hat{\rho}_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

 $d_i = \text{Rank}_{X_i} - \text{Rank}_{Y_i}$; distinct ranksKendall's τ

$$\hat{\tau} = \frac{n_c - n_d}{n(n-1)/2} = \frac{n_c}{n_c + n_d + n_t} - \frac{n_d}{n_c + n_d + n_t}$$

QTA 13

Simulation and Bootstrapping

3 formulas

Monte Carlo estimate

$$\hat{E}[g(X)] = \frac{1}{b} \sum_{i=1}^b g(X_i)$$

Sampling error

$$V[\hat{E}[g(X)]] = \frac{\sigma_g^2}{b} \quad \text{standard error } \frac{\sigma_g}{\sqrt{b}}$$

Antithetic variables

$$\text{standard error} = \frac{\sigma_g \sqrt{1 + \rho}}{\sqrt{b}}$$

smaller when $\rho < 0$

QTA 14

Machine-Learning Methods

5 formulas

Rescaling

$$\text{standardization } \tilde{x}_{ij} = \frac{x_{ij} - \mu_i}{\sigma_i} \quad \text{normalization } \tilde{x}_{ij} = \frac{x_{ij} - x_{i,\min}}{x_{i,\max} - x_{i,\min}}$$

Distances

$$\text{Euclidean } d_E = \sqrt{\sum_{i=1}^m (x_{iQ} - x_{iP})^2}$$

$$\text{Manhattan } d_M = \sum_{i=1}^m |x_{iQ} - x_{iP}|$$

K-means inertia

$$I = \sum_{j=1}^n d_j^2$$

Q-value update, Monte Carlo

$$Q_{\text{new}}(S, A) = Q_{\text{old}}(S, A) + \alpha [R - Q_{\text{old}}(S, A)]$$

 R = total subsequent reward

Q-value update, temporal difference

$$Q_{\text{new}}(S, A) = Q_{\text{old}}(S, A) + \alpha [R + V(S') - Q_{\text{old}}(S, A)]$$

 R = reward to the next decision; $V(S')$ = highest current Q-value in the next state

Logistic regression

$$p = \frac{1}{1 + \exp[-(\alpha + \beta_1 x_1 + \dots + \beta_k x_k)]} \quad \ln \frac{p}{1-p} = \alpha + \sum \beta_j x_j$$

Ridge and LASSO

$$\text{ridge: } \min \text{RSS} + \lambda \sum \beta_j^2 \quad \text{LASSO: } \min \text{RSS} + \lambda \sum |\beta_j|$$

LASSO can set coefficients to exactly 0

Mean squared forecast error

$$\text{MSFE} = \frac{1}{n_{\text{test}}} \sum_{i=1}^{n_{\text{test}}} (y_i - \hat{y}_i)^2$$

Confusion matrix

Formula

Accuracy

$$\frac{TP + TN}{TP + TN + FP + FN}$$

Precision

$$\frac{TP}{TP + FP}$$

Recall

$$\frac{TP}{TP + FN}$$

Error rate

$$\frac{FP + FN}{TP + TN + FP + FN} = 1 - \text{accuracy}$$

Decision tree impurity

$$\text{entropy} = - \sum_{i=1}^M p_i \log_2 p_i \quad \text{Gini} = 1 - \sum_{i=1}^M p_i^2$$

BOOK 3 OF 4 • 30% OF THE EXAM

Financial Markets and Products formulas

FMP 1

Banks

Concepts only

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

FMP 2

Insurance Companies and Pension Plans

8 formulas

Survival probability

$$P(\text{survive to } N+1) = P(\text{survive to } N) \times (1 - q_N)$$

 q_N = probability of death within the year from age N , given alive at N Survival from age X to age Y

$$P(X \rightarrow Y) = \frac{P(\text{survive from birth to } Y)}{P(\text{survive from birth to } X)}$$

Break-even premium

PV of expected premiums = PV of expected payouts

each premium weighted by the probability of being alive to pay it; each payout by the probability of death in that year

Ratio

Formula

Loss ratio

Payouts / premiums

Expense ratio

Expenses / premiums

Combined ratio

Loss ratio + expense ratio

Combined ratio after dividends

Combined ratio + dividends as % of premiums

Operating ratio

Combined ratio after dividends – investment income as % of premiums

Net asset value

$$NAV = \frac{\text{Fund assets} - \text{Fund liabilities}}{\text{Shares outstanding}}$$

Hedge fund return after fees

$$\text{investor return} = \text{gross} - \text{management fee} - \text{incentive fee}$$

$$\text{incentive fee} = \text{incentive rate} \times \max(0, \text{return after management fee} - \text{hurdle rate}),$$

charged only on value above the high-water mark

Forward payoff

$$\text{long } S_T - K \quad \text{short } K - S_T$$

option payoffs are in FMP 12

Futures margin

$$\text{variation margin} = \Delta F \times \text{contract size} \times \text{number of contracts}$$

margin call when the balance falls below the maintenance margin; top up to the initial margin

Buying stock on margin

$$\text{equity} = \text{market value} - \text{loan}$$

$$\text{margin call when } \frac{\text{equity}}{\text{market value}} < \text{maintenance ratio}$$

$$\text{trigger price } P^* = P_0 \frac{1 - \text{IM}}{1 - \text{MM}}$$

IM, MM = initial and maintenance margin ratios

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

Basis

$$b_t = S_t - F_t$$

Hedged price

$$\text{short hedge, price received} = F_0 + b_t \quad \text{long hedge, price paid} = F_0 + b_t$$

Minimum variance hedge ratio

$$h^* = \rho \frac{\sigma_S}{\sigma_F} \quad \text{hedge effectiveness} = \rho^2$$

Optimal number of contracts

$$N^* = h^* \frac{Q_A}{Q_F} \quad \text{tailing the hedge: } N^* = h^* \frac{V_A}{V_F} = h^* \frac{Q_A}{Q_F} \cdot \frac{S}{F}$$

Stock index hedge

$$N^* = \beta \frac{V_A}{V_F}$$

 V_F = futures price $\times$ multiplier

Changing portfolio beta

$$N^* = (\beta^* - \beta) \frac{V_A}{V_F}$$

$N^* < 0$: short futures to lower beta; $N^* > 0$: long futures to raise it

FMP 9

Foreign Exchange Markets

7 formulas

Real interest rate

$$R_{\text{real}} = \frac{1 + R_{\text{nom}}}{1 + R_{\text{infl}}} - 1 \approx R_{\text{nom}} - R_{\text{infl}}$$

Bid-ask spread

$$\text{spread} = \text{ask} - \text{bid} \quad \% \text{ spread} = \frac{\text{ask} - \text{bid}}{\text{mid}}$$

Covered interest parity

$$F = S \left(\frac{1 + R_{YYY}}{1 + R_{XXX}} \right)^T$$

rate quoted XXXYYY (units of YYY per XXX)

Forward premium and points

$$\frac{F - S}{S} \approx (R_{YYY} - R_{XXX}) T \quad \text{forward points} = (F - S) \times 10,000$$

Uncovered interest parity

$$E(S_T) = S_0 \left(\frac{1 + R_{YYY}}{1 + R_{XXX}} \right)^T$$

Purchasing power parity

$$\frac{S_T}{S_0} = \left(\frac{1 + \pi_{YYY}}{1 + \pi_{XXX}} \right)^T \quad \% \text{ change in } S \approx \pi_{YYY} - \pi_{XXX}$$

Appreciation and depreciation

$$\text{XXXYYY moves from } S_0 \text{ to } S_1: \text{XXX changes by } \frac{S_1 - S_0}{S_0}$$

$$\text{YYY changes by } \frac{S_0 - S_1}{S_1}$$

EXAM TIP The two percentage changes differ, so work out each currency on its own base.

FMP 10

Pricing Financial Forwards and Futures

9 formulas

Forward price

Annual compounding

Continuous

No income

$$S_0(1 + R)^T$$

$$S_0 e^{rT}$$

Known income, PV I

$$(S_0 - I)(1 + R)^T$$

$$(S_0 - I)e^{rT}$$

Known yield Q (index: dividend yield)

$$S_0 \left(\frac{1 + R}{1 + Q} \right)^T$$

$$S_0 e^{(r-Q)T}$$

Arbitrage

actual $F_0 >$ theoretical: buy the asset, short the forward

actual $F_0 <$ theoretical: short the asset, go long the forward

Asset

Value of a long forward

No income

$$S_0 - \frac{K}{(1 + R)^T}$$

Known income

$$(S_0 - I) - \frac{K}{(1 + R)^T}$$

Known yield

$$\frac{S_0}{(1 + Q)^T} - \frac{K}{(1 + R)^T}$$

Any asset

$$\frac{F_0 - K}{(1 + R)^T}; \text{ short} = - \text{ long}$$

Short sale profit,
dividend-paying stock

$S_0 - S_T - \text{dividends paid while short}$
plus interest on the proceeds if earned

FMP 11

Commodity Forwards and Futures

5 formulas

Forward with lease rate

$$F_0 = S_0 \left(\frac{1+R}{1+l} \right)^T \quad \text{implied } l = \left(\frac{S_0}{F_0} \right)^{1/T} (1+R) - 1$$

Storage cost and
convenience yield

$$F_0 = (S_0 + U) \left(\frac{1+R}{1+Y} \right)^T \quad \text{implied } Y = \left(\frac{S_0 + U}{F_0} \right)^{1/T} (1+R) - 1$$

U = PV of storage costs; convenience yield belongs to consumption commodities

No-arbitrage upper
bounds

$$F_0 \leq S_0(1+R)^T \quad \text{with storage: } F_0 \leq (S_0 + U)(1+R)^T$$

Continuous cost of carry

$$F_0 = S_0 e^{(c-y)T}$$

c = interest rate + storage cost rate, y = convenience yield

Expected future spot
price

$$E(S_T) = F_0 \frac{(1+X)^T}{(1+R)^T}$$

X = expected return on the commodity

FMP 12

Options Markets

4 formulas

Position	Payoff	Profit
Long call	$\max(S_T - K, 0)$	$\max(S_T - K, 0) - c_0$
Short call	$-\max(S_T - K, 0)$	$c_0 - \max(S_T - K, 0)$
Long put	$\max(K - S_T, 0)$	$\max(K - S_T, 0) - p_0$
Short put	$-\max(K - S_T, 0)$	$p_0 - \max(K - S_T, 0)$

FMP 13

Properties of Options

19 formulas

European call	Lower bound	Upper
No dividends	$\max(S_0 - \text{PV}(K), 0)$	S_0
Dividends	$\max(S_0 - \text{PV}(D) - \text{PV}(K), 0)$	S_0
Other assets	$\max(\text{PV}(F) - \text{PV}(K), 0)$	$\text{PV}(F)$
European put	Lower bound	Upper
No dividends	$\max(\text{PV}(K) - S_0, 0)$	$\text{PV}(K)$
Dividends	$\max(\text{PV}(D) + \text{PV}(K) - S_0, 0)$	$\text{PV}(K)$
Other assets	$\max(\text{PV}(K) - \text{PV}(F), 0)$	$\text{PV}(K)$
American call	Lower bound	Upper
No dividends	$\max(S_0 - \text{PV}(K), 0)$	S_0
Dividends	$\max(S_0 - K, S_0 - \text{PV}(D) - \text{PV}(K), 0)$	S_0

American put	Lower bound	Upper
No dividends	$\max(K - S_0, 0)$	K
Dividends	$\max(K - S_0, PV(D) + PV(K) - S_0, 0)$	K

Put-call parity, European

non-dividend $c + PV(K) = p + S_0$

dividend $c + PV(D) + PV(K) = p + S_0$

forward form $c + PV(K) = p + PV(F)$

Put-call relation, American

non-dividend $S_0 - K \leq C - P \leq S_0 - PV(K)$

dividend $S_0 - PV(D) - K \leq C - P \leq S_0 - PV(K)$

Early exercise of an American call

exercise just before an ex-dividend date can be optimal only if $D > K - K^*$

never exercise early without dividends

$K^* = K$ discounted from the next ex-dividend date (or maturity) back to this one

Increase in	European call	European put	American call	American put
S_0	+	-	+	-
K	-	+	-	+
T	?	?	+	+
σ	+	+	+	+
r	+	-	+	-
Dividends	-	+	-	+

FMP 14

Trading Strategies

10 formulas

Option and stock equivalences

protective put $S + P = C + PV(K)$

covered call $S - C = PV(K) - P$

Payoff at T	$S_T \leq K_1$	$K_1 < S_T \leq K_2$	$S_T > K_2$
Bull: long call K_1 , short call K_2	0	$S_T - K_1$	$K_2 - K_1$
Bear: long put K_2 , short put K_1	$K_2 - K_1$	$K_2 - S_T$	0
Box: bull call + bear put	$K_2 - K_1$	$K_2 - K_1$	$K_2 - K_1$
Strangle: long put K_1 , long call K_2	$K_1 - S_T$	0	$S_T - K_2$

Box spread value

$PV(K_2 - K_1)$

Butterfly (calls)

0 below K_1 $S_T - K_1$ from K_1 to K_2 $K_3 - S_T$ from K_2 to K_3

0 above K_3

long call K_1 , short 2 calls K_2 , long call K_3 ; $K_2 = (K_1 + K_3)/2$

Straddle

long call + long put, same K : payoff $|S_T - K|$

Strip and strap

strip = 1 call + 2 puts (bearish tilt) strap = 2 calls + 1 put (bullish tilt)

same strike and maturity

Principal protected note

zero-coupon bond paying L + call option

feasible if $L - Le^{-rT} \geq \text{option cost}$

FMP 15**Exotic Options**

11 formulas

Zero-cost call

payoff $\max(S_T - K - A, -A)$, $A = c(1 + R)^T$

Gap options

strike K_1 , trigger K_2 : call pays $S_T - K_1$ if $S_T > K_2$

put pays $K_1 - S_T$ if $S_T < K_2$ else 0

the payoff can be negative

Payoff	Call	Put
Average price	$\max(S_{\text{avg}} - K, 0)$	$\max(K - S_{\text{avg}}, 0)$
Average strike	$\max(S_T - S_{\text{avg}}, 0)$	$\max(S_{\text{avg}} - S_T, 0)$
Floating lookback	$S_T - S_{\min}$	$S_{\max} - S_T$
Fixed lookback	$\max(S_{\max} - K, 0)$	$\max(K - S_{\min}, 0)$
Cash-or-nothing	Q if $S_T > K$	Q if $S_T < K$
Asset-or-nothing	S_T if $S_T > K$	S_T if $S_T < K$

Barrier parity

knock-in + knock-out = vanilla option

same barrier, strike and maturity

Exchange option

payoff $\max(U_T - V_T, 0)$

receive asset U , give up asset V

Volatility and variance swaps

receive realized: $L_{\text{vol}}(\sigma - \sigma_K)$ $L_{\text{var}}(\sigma^2 - V_K)$

$V_K = \sigma_K^2$; $L_{\text{var}} = L_{\text{vol}}/(2\sigma_K)$; paying realized reverses the signs

FMP 16**Properties of Interest Rates**

10 formulas

Compounding

$FV = PV(1 + R/m)^{mT}$ $PV = FV(1 + R/m)^{-mT}$

continuous: $FV = PVe^{RT}$, $PV = FVe^{-RT}$

Converting rates

$$R_2 = m_2 \left[\left(1 + \frac{R_1}{m_1} \right)^{m_1/m_2} - 1 \right] \quad R_m = m (e^{R_c/m} - 1)$$

$$R_c = m \ln(1 + R_m/m)$$

Bond price from yield

$$B = \frac{c}{2} \sum_{i=1}^{2T} \left(1 + \frac{y}{2} \right)^{-i} + 100 \left(1 + \frac{y}{2} \right)^{-2T}$$

Bond price from spot rates

$$B = \sum c_i e^{-R_i t_i}, \text{ or } \sum \frac{c_i}{(1 + R_i/m)^{m t_i}}$$

Par yield

$$c = \frac{(100 - 100 d_T) m}{A}$$

d_T = final discount factor; A = sum of coupon-date discount factors

Duration

$$\text{Macaulay } D = \sum t_i \frac{PV_i}{B} \quad \text{modified } D_{\text{mod}} = \frac{D_{\text{Mac}}}{1 + y/m}$$

$$\text{dollar } D_{\$} = D_{\text{mod}} B$$

Convexity

$$C = \sum t_i^2 \frac{PV_i}{B}$$

Price change

$$\Delta B = -DB \Delta y + \frac{1}{2} CB (\Delta y)^2 \quad \text{with dollar duration: } \Delta B = -D_{\$} \Delta y$$

Forward rates

$$\text{discrete } F = \left[\frac{(1 + R_2)^{T_2}}{(1 + R_1)^{T_1}} \right]^{1/(T_2 - T_1)} - 1$$

$$\text{continuous } F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

FRA, pay fixed

$$\text{payoff at the start of the period } \frac{(R - R_K) \tau L}{1 + R\tau}$$

$$\text{value today} = (R_F - R_K) \tau L \text{ discounted from the period end}$$

receive fixed: signs reversed

FMP 17

Corporate Bonds

2 formulas

Expected loss and return

$$\text{expected loss rate} = \text{PD} \times (1 - \text{RR})$$

$$\text{expected return} = r_f + \text{credit spread} - \text{expected loss rate}$$

Default rates

$$\text{issue default rate} = \frac{\text{defaulted issues}}{\text{total issues}}$$

$$\text{dollar default rate} = \frac{\text{par value defaulted}}{\text{total par value}}$$

FMP 18

Mortgages and Mortgage-Backed Securities

6 formulas

Monthly mortgage payment

$$X = \frac{A (R/12)}{1 - (1 + R/12)^{-12T}}$$

A = amount borrowed

Interest and principal

$$\text{Interest}_t = \text{Balance}_{t-1} \times R/12 \quad \text{Principal}_t = X - \text{Interest}_t$$

$$\text{Balance}_t = \text{Balance}_{t-1} - \text{Principal}_t$$

WAC and WAM

$$\text{WAC} = \sum w_i c_i \quad \text{WAM} = \sum w_i L_i \quad w_i = \frac{P_i}{\sum P_i}$$

P_i = remaining principal of mortgage i

SMM and CPR

$$\text{SMM} = \frac{\text{prepayment in the month}}{\text{beginning balance} - \text{scheduled principal}}$$

$$\text{CPR} = 1 - (1 - \text{SMM})^{12}$$

Dollar roll value

$$A - B + C - D$$

A = sale price incl. AI; B = repurchase price incl. AI; C = interest on proceeds for the month; D = coupon and principal given up

Refinancing incentive

$$I = \text{WAC} - R$$

R = current mortgage rate

FMP 19**Interest Rate Futures**

10 formulas

Market	Day count
Treasury bonds (and AI on futures delivery)	Actual/actual
Corporate and municipal bonds	30/360
T-bills and money market	Actual/360
Accrued interest and dirty price	$\text{AI} = \frac{\text{days since last coupon}}{\text{days in coupon period}} \times \text{coupon}$ dirty = clean + AI
T-bill	$Q = \frac{360}{n}(100 - C) \quad C = 100 - \frac{Qn}{360}$ Q = quote, C = cash price per 100, n = days to maturity
Conversion factor	price per 1 of face of the delivered bond at a 6% yield (semiannual), maturity rounded down to the nearest 3 months
Delivery	cash received = settlement price $\times$ CF + AI CTD minimizes quoted price – settlement price $\times$ CF
Treasury futures price	$F_0 = (S_0 - I)(1 + R)^T, \text{ or } (S_0 - I)e^{rT}$ I = PV of coupons during the futures life
Duration-based hedge	$N^* = \frac{P D_P}{V_F D_F}$ P, D_P at hedge maturity; V_F = contract price
DV01 hedge	$N = -\frac{E_V}{E_F}$ E = value gain for a 1 bp fall in all rates; $N > 0$ long, $N < 0$ short

FMP 20**Swaps**

8 formulas

Comparative advantage total gain = $|\Delta_{\text{fixed}} - \Delta_{\text{floating}}|$

Swap position	Value
Pay fixed, receive floating	$B_{\text{float}} - B_{\text{fix}}$
Receive fixed, pay floating	$B_{\text{fix}} - B_{\text{float}}$
Currency: receive domestic (S_0 = domestic per foreign)	$B_D - S_0 B_F$
Currency: receive foreign	$S_0 B_F - B_D$

Bond values inside a swap

$$B_{\text{fix}} = \sum c \exp(-r_i t_i) + L \exp(-r_n t_n) \quad B_{\text{float}} = (L + k^*) \exp(-r^* t^*)$$

$$B_{\text{float}} = L \text{ immediately after a reset}$$

Swap value from FRAs

$$\text{pay fixed: } V = \sum \text{PV}[(R_{F,i} - R_K) \tau L]$$

$$R_{F,i} = \text{forward rate for period } i$$

Currency swap from forwards

$$\text{receive foreign: } V = \sum \text{PV}(\text{FCF}_i \times F_i - \text{DCF}_i)$$

FCF, DCF = foreign and domestic cash flows; F_i = forward rate (domestic per foreign);
receive domestic: sign reversed

BOOK 4 OF 4 • 30% OF THE EXAM

Valuation and Risk Models formulas

VRM 1 Measures of Financial Risk

3 formulas

Two-asset portfolio

$$\mu_P = w_1 \mu_1 + w_2 \mu_2 \quad \sigma_P = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 \rho w_1 w_2 \sigma_1 \sigma_2}$$

Normal distribution of

VaR at confidence X

ES at confidence X

Losses (μ_L, σ_L)

$$\mu_L + \sigma_L z$$

$$\mu_L + \sigma_L \frac{e^{-z^2/2}}{(1-X)\sqrt{2\pi}}$$

Returns (μ_P, σ_P)

$$\sigma_P z - \mu_P$$

$$\sigma_P \frac{e^{-z^2/2}}{(1-X)\sqrt{2\pi}} - \mu_P$$

EXAM TIP VaR from a loss distribution adds the mean; from a return distribution it subtracts the mean.

VRM 2 Calculating and Applying VaR

6 formulas

Linear portfolio

$$\Delta P = \sum q_i \Delta r_i$$

$$q_i = n_i S_i \text{ (amount invested), } \Delta r_i = \Delta S_i / S_i$$

Risk factor sensitivities

$$\text{linear } \Delta P = \sum \delta_i \Delta S_i \quad \text{delta-gamma } \Delta P = \sum \delta_i \Delta S_i + \frac{1}{2} \sum \gamma_i (\Delta S_i)^2$$

Zero-mean VaR and ES

$$\text{VaR} = \sigma_P z \quad \text{ES} = \sigma_P \frac{e^{-z^2/2}}{(1-X)\sqrt{2\pi}}$$

Square-root-of-time rule

$$\text{VaR}(T, X) = \sqrt{T} \text{VaR}(1, X) \quad \text{ES}(T, X) = \sqrt{T} \text{ES}(1, X)$$

needs iid returns and a zero mean

Delta-normal VaR of an option

$$\text{VaR} = |\delta| \times S \times \sigma \times z$$

Historical simulation

$$\text{VaR} = \text{the } n(1-X)\text{-th worst of the } n \text{ scenario losses}$$

$$\text{ES} = \text{average of the } n(1-X) \text{ worst losses}$$

Variance from daily returns

$$\sigma_n^2 = \frac{1}{m} \sum_{i=1}^m u_{n-i}^2$$

$$u_i = (S_i - S_{i-1})/S_{i-1}; \text{ mean return taken as 0}$$

EWMA

$$\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1 - \lambda) u_{n-1}^2$$

$$\text{RiskMetrics } \lambda = 0.94$$

GARCH(1,1)

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

$$\omega = \gamma V_L, \gamma = 1 - \alpha - \beta$$

Long-run variance

$$V_L = \frac{\omega}{1 - \alpha - \beta}$$

GARCH variance forecast

$$E[\sigma_{n+t}^2] = V_L + (\alpha + \beta)^t (\sigma_n^2 - V_L)$$

EWMA covariance and correlation

$$\text{cov}_n = \lambda \text{cov}_{n-1} + (1 - \lambda) x_{n-1} y_{n-1} \quad \rho_n = \frac{\text{cov}_n}{\sigma_{x,n} \sigma_{y,n}}$$

$$x, y = \text{returns on day } n - 1$$

Hazard rate

$$\text{PD}(0, t) = 1 - e^{-\bar{h}t} \quad \text{survival } S(t) = e^{-\bar{h}t}$$

Average hazard rate from survival

$$\bar{h} = -\frac{\ln S(t)}{t}$$

Default between t_1 and t_2

$$\exp(-\bar{h}_1 t_1) - \exp(-\bar{h}_2 t_2)$$

$$\bar{h}_2 = \text{average hazard rate to } t_2$$

Conditional PD

$$P(\text{default in year } t \mid \text{survived to } t-1) = \frac{S(t-1) - S(t)}{S(t-1)}$$

Expected loss

$$\text{EL} = \text{PD} \times \text{LGD} \times \text{EAD}$$

$$\text{LGD} = 1 - \text{RR}; \text{ in \% terms } \text{EL} = \text{PD} \times \text{LGD}$$

Rating transition matrix

$$n\text{-year matrix} = M^n$$

$$\text{assumes independence across years; cumulative PD is the default column}$$

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

Loss on one loan

$$\text{mean } p_i L_i (1 - R_i) \quad \text{SD } \sigma_i = \sqrt{p_i - p_i^2} L_i (1 - R_i)$$

Portfolio of identical loans

$$\sigma_P^2 = n\sigma^2 + n(n-1)\rho\sigma^2 \quad \frac{\sigma_P}{nL} = \frac{\sigma\sqrt{1+(n-1)\rho}}{L\sqrt{n}}$$

One-factor Gaussian copula

$$U_i = a_i F + \sqrt{1 - a_i^2} Z_i \quad \rho(U_i, U_j) = a_i a_j$$

Vasicek WCDR, 99.9%

$$\text{WCDR} = N\left(\frac{N^{-1}(\text{PD}) - \sqrt{\rho} N^{-1}(0.001)}{\sqrt{1 - \rho}}\right)$$

Credit risk capital

$$\text{worst-case loss} = \text{WCDR} \times \text{LGD} \times \text{EAD}$$

$$\text{capital} = \text{UL} = (\text{WCDR} - \text{PD}) \times \text{LGD} \times \text{EAD}$$

$$\text{loan by loan: } \sum (\text{WCDR}_i - \text{PD}_i) \text{LGD}_i \text{EAD}_i$$

Euler contribution

$$\text{Contribution}_i = x_i \frac{\partial F}{\partial x_i} \quad \sum_i \text{Contribution}_i = F$$

F = risk measure, x_i = size of loan i

VRM 7

Operational Risk

6 formulas

Basic indicator approach

$$\text{capital} = 15\% \times \text{average annual gross income over 3 years}$$

$$\text{gross income} = \text{interest earned} - \text{interest paid} + \text{non-interest income}$$

Standardized approach

$$\text{capital} = 3\text{-year average of } \sum \beta_k \text{GI}_k$$

β = 12% retail banking, asset management, retail brokerage; 15% commercial banking, agency services; 18% corporate finance, trading and sales, payment and settlement

SMA loss component

$$7X + 7Y + 5Z$$

X = average annual op loss; Y = from losses above €10m; Z = from losses above €100m (last 10 years)

Loss frequency (Poisson)

$$P(n) = \frac{e^{-\lambda} \lambda^n}{n!}$$

Scaling external loss data

$$\text{Loss}_A = \text{Loss}_B \left(\frac{\text{Revenue}_A}{\text{Revenue}_B} \right)^\beta$$

β = scaling exponent estimated from data

Power law

$$P(v > x) = Kx^{-\alpha}$$

VRM 8

Stress Testing

Concepts only

No formulas to memorise. GARP tests this chapter on concepts, so there is nothing from it to learn for this sheet.

VRM 9

Pricing Conventions, Discounting and Arbitrage

3 formulas

Present value

$$PV = \sum CF_t d_t$$

d_t = discount factor for time t

Money-market yield
from a discount quote

$$\frac{Qn/360}{100 - Qn/360}$$

360 to 365 day basis

$$r_{365} = r_{360} \times \frac{365}{360}$$

VRM 10

Interest Rates

4 formulas

Discount factors

$$\text{semiannual } d_t = (1 + z_t/2)^{-2t} \quad \text{continuous } d_t = e^{-z_t t}$$

Bond value

$$V = \frac{c}{2}A_T + 100d_T \quad V = 100 + \frac{c-p}{2}A_T$$

A_T = sum of discount factors; p = par rate

Six-month forward rate

$$1 + \frac{f}{2} = \frac{(1 + z_{T+0.5}/2)^{2T+1}}{(1 + z_T/2)^{2T}}$$

Spot rate from forwards

$$(1 + z_n/2)^n = (1 + f_1/2)(1 + f_2/2) \cdots (1 + f_n/2)$$

n = number of half-years

VRM 11

Bond Yields and Return Calculations

4 formulas

Annuity and perpetuity

$$\text{annuity } \frac{c}{y} [1 - (1 + y/2)^{-2T}] \quad \text{perpetuity } \frac{c}{y}$$

c = annual coupon, paid semiannually

Realized return

$$\text{gross } R = \frac{P_{\text{end}} + \text{coupons} - P_{\text{begin}}}{P_{\text{begin}}} \quad \text{net} = R - \frac{\text{financing cost}}{P_{\text{begin}}}$$

$$\text{several periods: } (1 + R_1) \cdots (1 + R_n) - 1$$

Spread

$$P = \sum_t \frac{CF_t}{[1 + (z_t + s)/2]^{2t}}, \text{ solve for } s$$

P&L decomposition

$$\text{P\&L} = \text{carry roll-down} + \text{rate changes} + \text{spread changes}$$

carry roll-down = price change from passing time plus coupon, on the assumed curve (forwards realized, or curve unchanged)

VRM 12

Applying Duration, Convexity and DV01

8 formulas

DV01

$$\text{DV01} = -\frac{\Delta P}{10,000 \times \Delta r}$$

Δr in decimal

Effective duration

$$D = \frac{P_- - P_+}{2P \Delta r} \quad \text{one-sided: } D = -\frac{1}{P} \frac{\Delta P}{\Delta r}$$

Effective convexity

$$C = \frac{P_+ + P_- - 2P}{P (\Delta r)^2}$$

Price change

$$\Delta P = -DP \Delta r + \frac{1}{2}CP (\Delta r)^2$$

Portfolio measures

$$\text{DV01}_P = \sum \text{DV01}_i \quad D_P = \sum \frac{V_i}{V} D_i \quad C_P = \sum \frac{V_i}{V} C_i$$

Hedge face amount

$$F_H = -F_P \frac{\text{DV01}_P}{\text{DV01}_H}$$

DV01s per 100 face

Duration and convexity hedge

$$\text{one bond: } P_H = -\frac{V D_V}{D_H} \quad \text{two bonds: } P_1 D_1 + P_2 D_2 = -V D_V$$

$$P_1 C_1 + P_2 C_2 = -V C_V$$

Barbell matching a bullet

$$V_S + V_L = V_{\text{bullet}} \text{ and } V_S D_S + V_L D_L = V_{\text{bullet}} D_{\text{bullet}}$$

the barbell has higher convexity

Key rate and bucket 01s $DV01 = \sum_i KR01_i$ also $DV01 = \sum$ forward bucket 01s

Portfolio SD from key rates

$$\sigma_P = \sqrt{\sum_i \sum_j \rho_{ij} \sigma_i \sigma_j KR01_i KR01_j}$$

σ_i = SD of daily change in key rate i (bp); ρ_{ij} = correlation of those changes

Portfolio SD from principal components

$$\sigma_P = \sqrt{\sigma_1^2 f_1^2 + \sigma_2^2 f_2^2 + \sigma_3^2 f_3^2}$$

f_i = value change per unit of factor i

Duration from a 01

$$\text{Duration} = \frac{10,000 \times 01 \text{ measure}}{\text{portfolio value}}$$

One step

$$f = e^{-rT} [pf_u + (1-p)f_d] \quad p = \frac{e^{rT} - d}{u - d} \quad u = e^{\sigma\sqrt{T}}, d = 1/u$$

$$S_u = S_0 u, S_d = S_0 d$$

Several steps

$$f = e^{-r\Delta t} [pf_u + (1-p)f_d] \quad u = e^{\sigma\sqrt{\Delta t}}, d = 1/u$$

American option node

$$f = \max(\text{exercise value}, e^{-r\Delta t} [pf_u + (1-p)f_d])$$

Delta

$$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}$$

Underlying

$$a \ln p = (a - d)/(u - d)$$

Non-dividend stock

$$e^{r\Delta t}$$

Dividend yield q or index

$$e^{(r-q)\Delta t}$$

Currency

$$e^{(r-r_f)\Delta t}$$

Futures

$$1$$

Lognormal stock price

$$\frac{\Delta S}{S} \sim N(\mu \Delta t, \sigma^2 \Delta t) \quad E(S_T) = S_0 e^{\mu T}$$

Distribution of $\ln S_T$

$$E(\ln S_T) = \ln S_0 + (\mu - \sigma^2/2)T \quad \text{SD} = \sigma\sqrt{T}$$

Realized return

$$R = \frac{1}{T} \ln \frac{S_T}{S_0} \sim N\left(\mu - \frac{\sigma^2}{2}, \frac{\sigma^2}{T}\right)$$

Historical volatility

$$u_i = \ln(S_i/S_{i-1}) \quad \hat{\sigma} = \frac{s}{\sqrt{\tau}}$$

s = sample SD of u_i ; τ = interval length in years

BSM, no dividends

$$c = S_0 N(d_1) - K e^{-rT} N(d_2) \quad p = K e^{-rT} N(-d_2) - S_0 N(-d_1)$$

d_1 and d_2

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad d_2 = d_1 - \sigma\sqrt{T}$$

Dividend yield q

$$c = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$$

$$d_1 = \frac{\ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$

same for a stock index

Currency options

$$q = r_f: c = S_0 e^{-r_f T} N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-r_f T} N(-d_1)$$

Futures options (Black)

$$c = e^{-rT} [F_0 N(d_1) - K N(d_2)] \quad p = e^{-rT} [K N(-d_2) - F_0 N(-d_1)]$$

$$d_1 = \frac{\ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

Discrete dividends

$$c = [S_0 - \text{PV}(D)] N(d_1) - K e^{-rT} N(d_2)$$

replace S_0 by $S_0 - \text{PV}(D)$ in every no-dividend formula, including d_1

Warrants

$$\text{value per warrant} = \frac{N}{N + M} \times c$$

$$\text{dilution cost to existing shareholders} = M \times \text{value per warrant}$$

c = BSM call on the stock; N = shares outstanding; M = warrants issued

VRM 16

The Greeks

23 formulas

Greek, no income	Call	Put
Delta	$N(d_1)$	$N(d_1) - 1$
Gamma	$\frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$	same as call
Vega	$S_0 \sqrt{T} N'(d_1)$	same as call
Rho	$K T e^{-rT} N(d_2)$	$-K T e^{-rT} N(-d_2)$

Theta, no income

$$\text{call} - \frac{S_0 N'(d_1) \sigma}{2\sqrt{T}} - r K e^{-rT} N(d_2)$$

$$\text{put} - \frac{S_0 N'(d_1) \sigma}{2\sqrt{T}} + r K e^{-rT} N(-d_2)$$

Standard normal density

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Greek, yield q	Call	Put
Delta	$e^{-qT} N(d_1)$	$e^{-qT} [N(d_1) - 1]$
Gamma	$\frac{N'(d_1) e^{-qT}}{S_0 \sigma \sqrt{T}}$	same as call
Vega	$S_0 \sqrt{T} N'(d_1) e^{-qT}$	same as call
Rho	$K T e^{-rT} N(d_2)$	$-K T e^{-rT} N(-d_2)$

Theta, yield q , call

$$-\frac{S_0 N'(d_1) \sigma e^{-qT}}{2\sqrt{T}} + q S_0 N(d_1) e^{-qT} - r K e^{-rT} N(d_2)$$

Theta, yield q , put
$$-\frac{S_0 N'(d_1) \sigma e^{-qT}}{2\sqrt{T}} - qS_0 N(-d_1)e^{-qT} + rKe^{-rT}N(-d_2)$$

 d_1 uses $r - q$, as in VRM 15

Currency and futures options use $q = r_f$ for currencies $q = r$ for futures

Long position	Delta	Gamma	Vega
Call	+	+	+
Put	-	+	+
Stock	1	0	0
Forward, non-dividend stock	1	0	0
Futures, non-dividend stock	e^{rT}	0	0
Forward, yield q	e^{-qT}	0	0
Futures, yield q	$e^{(r-q)T}$	0	0

EXAM TIP Short positions reverse every sign in the table above.

Portfolio Greeks
$$\Delta_P = \sum w_i \Delta_i \quad \Gamma_P = \sum w_i \Gamma_i \quad \nu_P = \sum w_i \nu_i$$

 w_i = number of options of type i

Gamma-neutral, then delta-neutral options to trade $n = -\frac{\Gamma_P}{\Gamma_T}$ then shares $= -(\Delta_P + n\Delta_T)$

Theta, delta and gamma $\Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma = r\Pi$ delta-neutral: $\Theta + \frac{1}{2}\sigma^2 S^2 \Gamma = r\Pi$

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