

FRM Part 2 Formula Sheet 2026 edition

1 Market Risk 2 Credit Risk 3 Operational Risk and Resilience 4 Liquidity and Treasury Risk 5 Risk Management and Investment Management

MARKET RISK MEASUREMENT AND MANAGEMENT

MR 1 Estimating Market Risk Measures

Historical simulation VaR and ES

VaR = the $[(1 - \alpha)n + 1]$ th highest loss (the $n(1 - \alpha)$ th is also accepted);

ES = average of the $(1 - \alpha)n$ highest losses

n observations, confidence α ; 1,000 obs at 95%: 51st highest loss

Normal VaR, P/L and L/P data

P/L data: $\alpha \text{VaR} = -\mu_{P/L} + \sigma_{P/L} z_\alpha$;

L/P data: $\alpha \text{VaR} = \mu_{L/P} + \sigma_{L/P} z_\alpha$

Normal VaR, arithmetic returns

$$\alpha \text{VaR} = -(\mu_r - \sigma_r z_\alpha) P_{t-1}; \quad r_t = \frac{P_t + D_t - P_{t-1}}{P_{t-1}}$$

Lognormal VaR

$\alpha \text{VaR} = P_{t-1} (1 - \exp[\mu_R - \sigma_R z_\alpha])$;

$R_t = \ln \frac{P_t + D_t}{P_{t-1}}$, normally distributed

ES as average of tail VaRs

ES $\approx$ average of the $n - 1$ VaRs at confidence

$\alpha + k(1 - \alpha)/n$, $k = 1, \dots, n - 1$

tail beyond α cut into n equal-probability slices;

$n = 10$ at 95%: VaRs at 95.5%, 96%, ..., 99.5%

Coherent risk measure from quantiles

$$M_\phi = \int_0^1 \phi(p) q_p dp;$$

ES: $\phi(p) = 0$ for $p < \alpha$, $\phi(p) = \frac{1}{1 - \alpha}$ for $p \geq \alpha$

Standard error of a quantile

$$\text{var}(q) = \frac{p(1-p)}{n[f(q)]^2}; \quad \text{se}(q) = \frac{\sqrt{p(1-p)/n}}{f(q)};$$

90% CI: $[q - 1.645 \text{se}(q), q + 1.645 \text{se}(q)]$

p = prob. of a loss above $q + h/2$; $f(q)$ = prob. mass in $q \pm h/2$;
 h = bin width

The CI multiplier is two-tailed (90% CI uses 1.645, 95% CI uses 1.96) and is separate from the z that gives q itself (95% VaR: $q = 1.645$).

QQ plot	Reading
Linear	data match the reference distribution
Intercept, slope	$\approx$ location, scale (≈ 0 and 1: standard normal)
Steeper in the tails than the middle	heavier tails than the reference
Isolated points off the line	outliers

MR 2 Non-Parametric Approaches

Bootstrapped HS

draw B resamples of size n with replacement;

VaR (ES) = mean of the B resample VaRs (ESs)

Age-weighted HS (BRW)

$$w(i) = \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}; \quad w(i) = \lambda w(i-1)$$

$\lambda \rightarrow 1$ gives equal weights $1/n$

Volatility-weighted HS (Hull and White)

$$r_{t,i}^* = \frac{\sigma_{T,i}}{\sigma_{t,i}} r_{t,i}$$

$\sigma_{T,i}$ = current forecast; $\sigma_{t,i}$ = forecast made for day t

Correlation-weighted HS

$$\bar{R} = \bar{A} A^{-1} R;$$

$$2 \times 2: a_{11} = 1, a_{12} = 0, a_{21} = \rho, a_{22} = \sqrt{1 - \rho^2}$$

$A, \bar{A}$ = Choleski factors of the historical and current correlation matrices

MR 3 Parametric Approaches: Extreme Value

GEV VaR

$$\text{Fréchet } (\xi > 0): \text{VaR} = \mu_n - \frac{\sigma_n}{\xi} [1 - (-n \ln \alpha)^{-\xi}];$$

$$\text{Gumbel } (\xi = 0): \text{VaR} = \mu_n - \sigma_n \ln[-n \ln \alpha]$$

α = VaR confidence level; n = block size

POT VaR

$$\text{VaR} = u + \frac{\beta}{\xi} \left\{ \left[\frac{n}{N_u} (1 - \alpha) \right]^{-\xi} - 1 \right\}$$

u = threshold; N_u = number of excesses over u ; β = scale

POT ES

$$\text{ES} = \frac{\text{VaR}}{1 - \xi} + \frac{\beta - \xi u}{1 - \xi}, \quad \xi < 1$$

Tail index	Distribution	Tails
$\xi > 0$	Fréchet	heavy (t , Pareto, Lévy); returns usually $0 < \xi < 0.35$
$\xi = 0$	Gumbel	exponential (normal, lognormal)
$\xi < 0$	Weibull	lighter than normal

MR 4 Backtesting VaR

Exceptions as binomial

failure rate = N/T ; $f(x) = {}^T C_x p^x (1-p)^{T-x}$;

$$E(x) = pT; \quad V(x) = p(1-p)T$$

$$p = 1 - c$$

Normal approximation test

$$z = \frac{x - pT}{\sqrt{p(1-p)T}} \approx N(0, 1);$$

two-tailed 95% test: reject if $|z| > 1.96$

Kupiec unconditional coverage

$$\text{LR}_{uc} = -2 \ln[(1-p)^{T-N} p^N] + 2 \ln\{[1 - (N/T)]^{T-N} (N/T)^N\} \sim \chi^2(1)$$

reject if $\text{LR}_{uc} > 3.841$ (95%)

Conditional coverage

$$\text{LR}_{cc} = \text{LR}_{uc} + \text{LR}_{ind} \sim \chi^2(2); \quad \text{reject if } \text{LR}_{cc} > 5.991;$$

reject independence alone if $\text{LR}_{ind} > 3.841$

Exceptions ($T = 250, 99\%$)	Zone	Increase in k (from 3)
0 to 4	Green	+0.00
5	Yellow	+0.40
6	Yellow	+0.50
7	Yellow	+0.65
8	Yellow	+0.75
9	Yellow	+0.85
10 or more	Red	+1.00

The test confidence level (cutoffs 1.96, 3.841) is chosen separately from the VaR level c , and a Type I error rejects a correct model (5 or more exceptions: 10.8% at 99%, $T = 250$).

MR 5 VaR Mapping

General and specific risk

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i; \quad \beta_p = \sum w_i \beta_i;$$

$$V(R_p) = \beta_p^2 V(R_m) + \sum w_i^2 \sigma_{\varepsilon_i}^2$$

first term general (market) risk, second term specific risk

Principal and duration mapping

principal: $\text{VaR} = \text{PV} \times \text{VaR\% of the zero at the average maturity};$

duration: $\text{VaR} = \text{PV} \times \text{VaR\% of the zero at the portfolio duration}$

VaR% interpolated linearly between vertices

Cash-flow mapping VaR

$$\text{undiversified VaR} = \sum |x_i| V_i;$$

$$\text{diversified VaR} = \alpha \sqrt{x' \Sigma x} = \sqrt{(x \times V)' R (x \times V)};$$

$$\text{two vertices: } \sqrt{\text{VaR}_1^2 + \text{VaR}_2^2 + 2\rho \text{VaR}_1 \text{VaR}_2};$$

$$\text{VaR}_i = |x_i| \times V_i$$

x = PV of cash flows at each vertex; V = VaR% of each vertex;

R = correlation matrix

Tracking error VaR

$$\text{TE-VaR} = \alpha \sqrt{(x - x_0)' \Sigma (x - x_0)};$$

$$\text{variance improvement} = 1 - \left(\frac{\text{TE-VaR}}{\text{VaR of index}} \right)^2$$

x, x_0 = portfolio and benchmark exposures

Mapping a currency forward

$$f_t = S_t e^{-r^* \tau} - K e^{-r \tau} = (F_t - K) e^{-r \tau};$$

long forward = long foreign currency spot ($S e^{-r^* \tau}$) + long

foreign currency bill ($S e^{-r^* \tau}$) + short domestic bill ($K e^{-r \tau}$)

r^* = foreign rate

Mapping a commodity forward

exposure = $Q \times F \times e^{-r \tau}$ (PV of the forward position);

VaR = exposure $\times$ VaR% of the contract

Mapping an FRA

long $\tau_1 \times \tau_2$ FRA = long τ_1 bill + short τ_2 bill;

$$\text{each leg } x = \frac{N}{1 + R_1 \tau_1};$$

$$(1 + R_2 \tau_2) = (1 + R_1 \tau_1)[1 + F_{1,2}(\tau_2 - \tau_1)]$$

Mapping an interest rate swap

pay fixed, receive floating = long FRN + short fixed-coupon bond;

equivalently a portfolio of forward contracts

FRN: cash at reset; after reset, a bill to the next reset

Mapping an option (delta-normal)

$$\Delta = e^{-r^* \tau} N(d_1); \quad dc = x_1 \frac{dS}{S} + x_2 \frac{dP^*}{P^*} + x_3 \frac{dP}{P};$$

$$x_1 = x_2 = S e^{-r^* \tau} N(d_1), \quad x_3 = -K e^{-r \tau} N(d_2)$$

$$P^*, P = \text{foreign and domestic bills}$$

MR 7 Beyond Exceedance-Based Backtesting

Probability integral transform

$p_{t+1} = F_t(\text{PL}_{t+1})$; accurate model: PITs i.i.d. $U(0, 1)$

F_t = forecast CDF of P&L, evaluated at the realized P&L

PIT histogram	Model
Flat	accurate
Hump in the middle	too wide, conservative
Spikes at both ends (U shape)	too narrow, aggressive, tails too thin
Excess mass near 0	tail losses understated

MR 8 Correlation Basics

Realized correlation

$$\rho_{\text{realized}} = \frac{2}{n^2 - n} \sum_{i>j} \rho_{i,j}; \quad \text{number of pairs} = n(n-1)/2$$

Correlation swap payoff

fixed-rate payer (buys correlation): $N(\rho_{\text{realized}} - \rho_{\text{fixed}})$;

fixed-rate receiver: the negative

VaR with correlated assets

$$\text{VaR}_P = \sigma_P \alpha \sqrt{x}; \quad \sigma_P = \sqrt{\beta_h' C \beta_v};$$

$$\text{two assets: } \sigma_P = \sqrt{\beta_1^2 \sigma_1^2 + \beta_2^2 \sigma_2^2 + 2\rho \beta_1 \beta_2 \sigma_1 \sigma_2}$$

β = amounts invested; σ daily; x = days; C = covariance matrix;

α = normal quantile

Joint default probability

$$P(X \cap Y) = \rho_{XY} \sqrt{P_X(1 - P_X)} \sqrt{P_Y(1 - P_Y)} + P_X P_Y$$

Exchange option volatility

$$\sigma_E = \sqrt{\sigma_A^2 + \sigma_B^2 - 2 \text{Cov}_{AB}}$$

Option	Higher price with
Better of two, call on max, exchange, spread, better of two or cash, dual-strike	lower correlation
Worse of two, basket	higher correlation
Quanto call	lower $\rho(\text{asset}, \text{FX})$

MR 9 Empirical Properties of Correlation

Mean reversion model

$$S_t - S_{t-1} = a(\mu_S - S_{t-1})\Delta t + \sigma_S \varepsilon \sqrt{\Delta t};$$

$$\Delta t = 1, \text{ no noise: } E[S_t] = S_{t-1} + a(\mu_S - S_{t-1})$$

Mean reversion rate from regression

regress $S_t - S_{t-1}$ on S_{t-1} : $S_t - S_{t-1} = a\mu_S - aS_{t-1}$;

slope $\beta = -a$, so $a = -\beta$; intercept = $a\mu_S$

Autocorrelation

one-period autocorrelation = $1 - a$

mean reversion + autocorrelation = 1

Best-fit correlation distributions

equity: Johnson SB;

bond: generalized extreme value (normal also fits quite well);

default probability: Johnson SB

MR 10 Financial Correlation Modeling: Bottom-Up

Joint default probability, two assets

$$Q(t_B \leq t_1 \cap t_{Caa} \leq t_2) = M_2(x_B \leq N^{-1}(Q_B(t_1)) \cap x_{Caa} \leq N^{-1}(Q_{Caa}(t_2)); \rho);$$

n assets: M_n with correlation matrix ρ_M
 $M_n = n$ -variate standard normal CDF

Correlated default time of asset i

$$M_n(\cdot) = Q_i(\tau_i) \Rightarrow \tau_i = Q_i^{-1}(M_n(\cdot));$$

repeat over many draws from M_n and average τ_i

MR 11 Regression Hedging and PCA

DV01-neutral hedge

$$F^{30} = -F^{JNJ} \times \frac{DV01^{JNJ}}{DV01^{30}} \text{ (risk weight 100\%)}$$

Regression hedge

$$\Delta y_t^{JNJ} = \alpha + \beta \Delta y_t^{30} + \varepsilon_t; \quad F^{30} = -F^{JNJ} \times \frac{DV01^{JNJ}}{DV01^{30}} \times \hat{\beta};$$

risk weight: $\frac{-F^{30} DV01^{30}}{F^{JNJ} DV01^{JNJ}} = \hat{\beta}$

SD of regression-hedged P&L

$$\sigma_{P\&L} = \left| \frac{F^{JNJ} \times DV01^{JNJ}}{100} \right| \times \sigma_\varepsilon$$

σ_ε = standard error of the regression

Two-variable regression hedge

$$\Delta y_t^{20} = \alpha + \beta^{10} \Delta y_t^{10} + \beta^{30} \Delta y_t^{30} + \varepsilon_t;$$

$$F^{10} = -F^{20} \times \frac{DV01^{20}}{DV01^{10}} \times \hat{\beta}^{10};$$

$$F^{30} = -F^{20} \times \frac{DV01^{20}}{DV01^{30}} \times \hat{\beta}^{30};$$

risk weights: $\frac{-F^{10} DV01^{10}}{F^{20} DV01^{20}} = \hat{\beta}^{10}, \quad \frac{-F^{30} DV01^{30}}{F^{20} DV01^{20}} = \hat{\beta}^{30}$

Reverse regression hedge

regress Treasury on JNJ (slope $\hat{\beta}_{rev}$): risk weight of the hedge = $1/\hat{\beta}_{rev}$ (not $\hat{\beta}$)

Level regression $y_t = \alpha + \beta x_t + \varepsilon_t$ vs change regression

$\Delta y_t = \beta \Delta x_t + \Delta \varepsilon_t$: level errors are usually serially correlated ($\varepsilon_t = \rho \varepsilon_{t-1} + v_t$), so OLS stays unbiased and consistent but may not be efficient.

MR 12 Arbitrage Pricing with Term Structure Models

Binomial tree pricing

$$V_0 = \frac{p V_u + (1-p) V_d}{1+r/2} \text{ (six-month steps, semiannual rates);}$$

general step Δt : divide by $(1+r\Delta t)$

Replicating portfolio

$$F^{.5} + P_u(1) F^1 = V_u; \quad F^{.5} + P_d(1) F^1 = V_d;$$

option price = $P_0(.5) F^{.5} + P_0(1) F^1$
 F = face amounts of the six-month and one-year zeros;
 $P_u(1), P_d(1)$ = date 1 prices of the one-year zero

Risk-neutral probability and drift

solve $P_0 = \frac{p P_u + (1-p) P_d}{1+r/2}$ for p ;

risk-neutral drift = $p(r_u - r_0) + (1-p)(r_d - r_0)$

CMT swap

$$\text{payoff each six months} = \text{Notional} \times \frac{y_{CMT} - K}{2};$$

$$\text{node value} = \frac{p V_u + (1-p) V_d}{1+r/2} + \text{payoff at that node}$$

Option-adjusted spread

market price = model value with every discount rate r replaced by $r + \text{OAS}$ (cash flows still from unshifted rates);

OAS > 0: cheap, OAS < 0: rich;

$$E[dP/P] = (r_0 + \text{OAS}) dt$$

MR 13 Expectations, Risk Premium, and Convexity

Jensen's inequality and convexity effect

$$E\left[\frac{1}{1+r}\right] > \frac{1}{E[1+r]} = \frac{1}{1+E[r]} \Rightarrow f < E[r];$$

$$P(2) = \frac{\frac{1}{2} \left[\frac{1}{1+r_u} + \frac{1}{1+r_d} \right]}{1+r_0} = \frac{1}{(1+r_0)(1+f)};$$

convexity effect = $E[r] - f$, rises with maturity and volatility
 f = forward rate

Bond return decomposition

$$\frac{dP}{P} = f(T) dt - D dr + \frac{1}{2} C \sigma^2 dt;$$

$$E\left[\frac{dP}{P}\right] = f(T) dt - D E[dr] + \frac{1}{2} C \sigma^2 dt$$

$$D = -\frac{1}{P} \frac{\partial P}{\partial r}, \quad C = \frac{1}{P} \frac{\partial^2 P}{\partial r^2}$$

Expected return, risk-averse investor

risk-neutral: $E[dP/P] = r_0 dt$;

risk-averse: $E[dP/P] = r_0 dt + \lambda D dt$; SR = λ/σ

The forward rate splits into expectations, risk premium and convexity:
 $f(T) = \{r_0 + E[dr/dt] D\} + \lambda D - \frac{1}{2} C \sigma^2$.

MR 14 Term Structure Models: Drift

Model 1 (no drift)

$$dr = \sigma dw; \quad E[dr] = 0; \quad \text{SD}(dr) = \sigma \sqrt{dt};$$

tree: $r_0 \pm \sigma \sqrt{dt}$, then $r_0 \pm 2\sigma \sqrt{dt}$ and r_0 , probabilities $\frac{1}{2}$
 σ = annual basis-point volatility

Model 2 (constant drift)

$$dr = \lambda dt + \sigma dw;$$

tree: $r_0 + \lambda dt \pm \sigma \sqrt{dt}$, then $r_0 + 2\lambda dt \pm 2\sigma \sqrt{dt}$ and $r_0 + 2\lambda dt$;

after T years: mean $r_0 + \lambda T$, SD $\sigma \sqrt{T}$

Ho-Lee (time-dependent drift)

$$dr = \lambda_t dt + \sigma dw;$$

tree: $r_0 + \lambda_1 dt \pm \sigma \sqrt{dt}$, then $r_0 + (\lambda_1 + \lambda_2) dt \pm 2\sigma \sqrt{dt}$ and $r_0 + (\lambda_1 + \lambda_2) dt$

Vasicek rate change

$$dr = k(\theta - r) dt + \sigma dw; \quad \theta = r_\infty + \lambda/k;$$

$$E[dr] = k(\theta - r) dt, \quad \text{SD}(dr) = \sigma \sqrt{dt}$$

Vasicek in T years, half-life

$$E[r_T] = r_0 e^{-kT} + \theta(1 - e^{-kT}); \quad \text{SD} = \sqrt{\frac{\sigma^2}{2k}(1 - e^{-2kT})};$$

$$\text{half-life } \tau = \frac{\ln 2}{k}$$

dw has mean 0 and SD $\sqrt{dt}$: plug a given realization of dw straight into dr , and from a standard normal Z use $dw = Z \sqrt{dt}$.

MR 15 Term Structure Models: Volatility

Model 3 (time-dependent volatility)

$$dr = \lambda(t) dt + \sigma e^{-\alpha t} dw; \quad \text{SD of } dr \text{ at time } t: \sigma e^{-\alpha t} \sqrt{dt}$$

terminal SD equals the Vasicek SD with $k = \alpha$

CIR and lognormal rate change

$$\text{CIR: } dr = k(\theta - r) dt + \sigma \sqrt{r} dw;$$

$$\text{Model 4 (lognormal): } dr = ar dt + \sigma r dw;$$

$$\text{Courtadon: } dr = k(\theta - r) dt + \sigma r dw$$

Basis-point volatility

$$\text{normal: } \sigma; \quad \text{CIR: } \sigma \sqrt{r}; \quad \text{lognormal: } \sigma r;$$

$$\text{e.g. } \sigma_{\text{CIR}} \sqrt{8\%} = 1\% \Rightarrow \sigma_{\text{CIR}} = 3.54\%;$$

$$\sigma_y \times 8\% = 1\% \Rightarrow \sigma_y = 12.5\%$$

$$\text{lognormal } \sigma = \text{yield volatility, \% of the rate}$$

MR 16 The Vasicek and Gauss+ Models

Gauss+ factor changes

$$dr_t = -\alpha_r(r_t - m_t) dt;$$

$$dm_t = -\alpha_m(m_t - l_t) dt + \sigma_m(\rho dW_t^1 + \sqrt{1 - \rho^2} dW_t^2);$$

$$dl_t = -\alpha_l(l_t - \mu) dt + \sigma_l dW_t^1;$$

$$E[dW_t^1 dW_t^2] = 0, \quad dW = Z \sqrt{dt}$$

MR 17 Volatility Smiles and Surfaces

Put-call parity and implied volatility

$$p + S_0 e^{-qT} = c + K e^{-rT};$$

$$p_{\text{BS}} - p_{\text{mkt}} = c_{\text{BS}} - c_{\text{mkt}} \Rightarrow \text{implied vol of a European call} =$$

$$\text{implied vol of a European put (same } K, T)$$

$$\text{FX: } q = \text{foreign risk-free rate}$$

Options	Implied vol	Implied distribution
FX	smile: higher away from ATM	heavier both tails, more peaked
Equity	skew: falls as K rises	heavier left tail, thinner right tail
Single large jump expected	frown: ATM highest	bimodal

MR 18 Fundamental Review of the Trading Book

97.5% ES vs 99% VaR (normal losses)

$$\text{VaR}_{99\%} = \mu + 2.326\sigma; \quad \text{ES}_{97.5\%} = \mu + 2.338\sigma$$

$$\text{stressed 97.5\% ES replaces 99\% VaR;}$$

$$\text{liquidity horizons replace the 10-day horizon}$$

Liquidity horizon (days)	Level	Volatility
Interest rate	10 to 60	60
Equity, large cap	10	20
Equity, small cap	20	60
FX rate	10 to 40	40
Energy price	20	60
Precious metal price	20	60
Other commodities price	60	120

Liquidity horizon, other	Days
Credit spread: sovereign IG	20
Credit spread: sovereign non-IG	40
Credit spread: corporate IG	40
Credit spread: corporate non-IG	60
Credit spread: other	120
Credit spread volatility	120
Equity other	60
Commodity other	120

Liquidity-adjusted ES

$$\text{ES} = \sqrt{\text{ES}_1^2 + \sum_{j=2}^5 \left(\text{ES}_j \sqrt{\frac{\text{LH}_j - \text{LH}_{j-1}}{10}} \right)^2}$$

$$= \sqrt{\text{ES}_1^2 + \text{ES}_2^2 + 2\text{ES}_3^2 + 2\text{ES}_4^2 + 6\text{ES}_5^2}$$

$$\text{LH} = 10, 20, 40, 60, 120;$$

$$\text{ES}_j = \text{ES with 10-day shocks to categories } j \text{ and above only}$$

Backtesting and P&L attribution

backtest of 1-day VaR over 12 months: more than 12 exceptions at 99% or more than 30 at 97.5% $\Rightarrow$ standardized approach;

PLA ratios: $\text{mean}(U)/\text{SD}(V)$ within -10% to $+10\%$ and $\text{var}(U)/\text{var}(V) < 20\%$;

four or more PLA ratio breaches in 12 months $\Rightarrow$ standardized approach

$$U = \text{actual minus model P\&L}; \quad V = \text{actual P\&L}$$

Under the IMA, jump-to-default risk uses a 1-year, 99.9% VaR, while credit spread risk sits in ES with liquidity horizons of 20 to 120 days.

CREDIT RISK MEASUREMENT AND MANAGEMENT

CR 3 Credit Risk Management

Expected loss

$$\text{EL} = \text{PD} \times \text{EAD} \times \text{LGD}$$

$$\text{EL in currency; PD, LGD in \%}$$

IFRS 9 stage	Impairment	Interest on
1: performing	12-month ECL	Gross amount
2: arrears or significant credit deterioration	Lifetime ECL	Gross amount
3: nonperforming	Lifetime ECL	Net (carrying) amount

CR 4 Capital Structure in Banks

Expected loss, horizon H

$$\text{EL}_H = \text{EA}_H - E(\text{EA}_H) = \text{PD}_H \cdot \text{EA}_H \cdot \text{LR}_H;$$

$$\text{EL}_P = \sum \text{EL}_i = \sum \text{EA}_i \cdot \text{PD}_i \cdot \text{LR}_i$$

$$\text{EA} = \text{exposure amount, LR} = \text{loss rate (LGD)}$$

Standalone UL

$$\text{UL} = \text{EA} \cdot \sqrt{\text{PD} \cdot \sigma_{\text{LR}}^2 + \text{LR}^2 \cdot \sigma_{\text{PD}}^2}$$

EA, PD, LR independent

Variance of default

$$\sigma_{\text{PD}}^2 = \text{PD} (1 - \text{PD})$$

Portfolio UL

$$UL_P = \sqrt{\sum_i \sum_j \rho_{ij} UL_i UL_j};$$

$$\text{two loans: } UL_P = \sqrt{UL_1^2 + UL_2^2 + 2\rho_{12} UL_1 UL_2}$$

ρ_{ij} = default correlation over the horizon

UL contribution

$$ULC_i = \frac{UL_i \sum_j \rho_{ij} UL_j}{UL_P}; \quad \sum_i ULC_i = UL_P$$

Homogeneous portfolio

$$UL_P = UL_i \sqrt{n + \rho(n^2 - n)}; \quad \text{large } n: ULC_i \approx UL_i \sqrt{\rho}$$

n similar loans, common ρ

Economic capital

$$EC_P = UL_P \times CM; \quad EC_i = ULC_i \times CM$$

CM = capital multiplier

CR 5 Credit Risk Modeling and Assessment

Capital adequacy ratio

$$CAR = \frac{\text{Capital}}{\text{Risk-weighted assets}} > \alpha;$$

$$\text{Capital} = \text{Tier 1} + \text{Tier 2}; \quad \alpha = 8\% \text{ (Basel II);}$$

Basel III: $CAR \geq 10.5\%$

Risk-weighted assets

$$IRB: RWA = K \times 12.5 \times EAD;$$

$$\text{standardized: } RWA = \sum \text{risk weight} \times \text{exposure}$$

$$K = \text{capital requirement (function of PD, LGD, } M, R);$$

standardized uses prescribed supervisory risk weights

Merton PD and DD, real world

$$PD = N\left[-\frac{\ln(A/L) + (\mu - \sigma_A^2/2)T}{\sigma_A \sqrt{T}}\right];$$

$$DD = \frac{\ln(A/L) + (\mu - \sigma_A^2/2)T}{\sigma_A \sqrt{T}}$$

$$A = \text{asset value}, L = \text{face value of debt}, \mu = \text{expected asset return}$$

KMV default point

$$\text{Default point} = \text{short-term debt} + \frac{1}{2} \times \text{long-term debt}$$

RAROC

$$RAROC = \frac{\text{Loan revenues}}{\text{Capital at risk}};$$

$$\text{Loan revenues} = (s + f - l - c)(1 - x) \times \text{loan value};$$

accept if $RAROC > \text{cost of capital}$

$$s = \text{spread}, f = \text{fees}, l = \text{expected loan loss}, c = \text{operating cost}, x = \text{tax rate}$$

RAROC capital at risk

$$\text{Duration approach: } \Delta L = -L \cdot D \cdot \frac{\Delta i}{1 + i};$$

$$\text{default-data approach: unexpected loan loss} = \alpha \times LGD \times EAD$$

$$\alpha = 2.6\sigma \text{ at } 99.5\% \text{ if normal; } 5 \text{ to } 6 \text{ for skewed losses}$$

CR 7 Credit Scoring and Retail Credit Risk

Accuracy ratio

$$AR = \frac{A_R}{A_P}; \quad \text{closer to 1 is better}$$

$$A_R = \text{area between actual CAP and random } 45^\circ \text{ line};$$

A_P = same for the perfect model

Implied PD of a score band

$$\text{Implied PD} = \frac{\text{Loss rate}}{LGD}$$

CR 8 Sovereign Default Risk

Sovereign default spread

$$\text{Default spread} = \text{yield on sovereign bond in a currency} - \text{yield on riskless (e.g., US Treasury) bond in the same currency}$$

CR 9 Estimating Default Probabilities

Altman Z-score

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 0.999X_5;$$

$$Z > 3.0: \text{unlikely to default}; \quad 2.7 \text{ to } 3.0: \text{on alert};$$

$$1.8 \text{ to } 2.7: \text{good chance of default}; \quad Z < 1.8: \text{very high}$$

$$X_1 = \text{working capital/TA}, X_2 = \text{retained earnings/TA}, X_3 = \text{EBIT/TA}, X_4 = \text{market value of equity/book value of total liabilities}, X_5 = \text{sales/TA}$$

Marginal and conditional PD

$$\text{Unconditional PD in year } n = Q(n) - Q(n-1);$$

$$\text{conditional PD in year } n = \frac{Q(n) - Q(n-1)}{1 - Q(n-1)}$$

$$Q = \text{cumulative PD}$$

Hazard rate, survival, cumulative PD

$$V(t) = e^{-\int_0^t \lambda(\tau) d\tau}; \quad Q(t) = 1 - e^{-\bar{\lambda}(t)t};$$

$$\bar{\lambda}(t) = -\frac{1}{t} \ln[1 - Q(t)]$$

$$V = \text{survival probability}, \bar{\lambda} = \text{average hazard rate}$$

Average hazard rate from a spread

$$\bar{\lambda} = \frac{s(T)}{1 - R}; \quad \text{hazard between } T_1 \text{ and } T_2: \frac{\bar{\lambda}_2 T_2 - \bar{\lambda}_1 T_1}{T_2 - T_1}$$

s = CDS, bond yield or asset swap spread

CDS-bond basis

$$\text{CDS-bond basis} = \text{CDS spread} - \text{bond yield spread}$$

Merton model

$$E_0 = V_0 N(d_1) - D e^{-rT} N(d_2);$$

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}}, \quad d_2 = d_1 - \sigma_V \sqrt{T};$$

$$\sigma_E E_0 = N(d_1) \sigma_V V_0; \quad \text{debt} = V_0 - E_0;$$

$$\text{expected loss on debt} = \frac{D e^{-rT} - (V_0 - E_0)}{D e^{-rT}}$$

$$V_0 = \text{firm value}, D = \text{face value of debt}$$

Merton PD and DD, risk neutral

$$PD = N(-d_2); \quad DD = d_2 = \frac{\ln V_0 - \ln D + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}}$$

Risk-neutral PD (from spreads, $N(-d_2)$) exceeds real-world PD (historical); use risk-neutral PD for valuation and real-world PD for scenario analysis and credit VaR.

CR 10 Credit Value at Risk

Transition matrix, other horizons

$$n\text{-year matrix} = (\text{one-year matrix})^n;$$

$$\text{matrix for } 1/n \text{ of a year} = n\text{th root of the one-year matrix}$$

assumes independent periods

Vasicek WCDR and credit loss

$$WCDR(T, X) = N\left[\frac{N^{-1}(PD) + \sqrt{\rho} N^{-1}(X)}{\sqrt{1 - \rho}}\right];$$

$$\text{loss at } X \approx \sum_i WCDR_i(T, X) \times EAD_i \times LGD_i$$

$$\text{Basel IRB: } T = 1 \text{ year}, X = 99.9\% \text{ (times maturity adjustment)}$$

CreditRisk+ number of defaults

$$\text{Prob}(m \text{ defaults}) = \frac{e^{-qn} (qn)^m}{m!}$$

n loans, each with PD q

CreditMetrics rating thresholds

Sample $x \sim N(0, 1)$, correlated across obligors by equity correlation;
new rating boundaries at N^{-1} of cumulative transition probabilities from the best rating;
default if $x > N^{-1}(1 - \text{PD})$

Here credit VaR is the credit loss over T not exceeded with confidence X (the loss quantile itself, EL not deducted), while CR 11 deducts EL.

CR 11 Portfolio Credit Risk

Default correlation of two credits

$$\rho_{12} = \frac{\pi_{12} - \pi_1 \pi_2}{\sqrt{\pi_1(1 - \pi_1)} \sqrt{\pi_2(1 - \pi_2)}};$$

$$\pi_{12} = \rho_{12} \sqrt{\pi_1(1 - \pi_1)} \sqrt{\pi_2(1 - \pi_2)} + \pi_1 \pi_2;$$

$$P[1 \text{ or } 2 \text{ or both default}] = \pi_1 + \pi_2 - \pi_{12}$$

π = PD, π_{12} = joint PD

Single-factor model

$$a_i = \beta_i m + \sqrt{1 - \beta_i^2} \varepsilon_i; \quad \text{corr}(a_i, a_j) = \beta_i \beta_j;$$

default if $a_i \leq k_i$, $\pi_i = \Phi(k_i)$
 m = market factor, k_i = default threshold

Conditional default probability

Given m : a_i normal, mean $\beta_i m$, variance $1 - \beta_i^2$;

$$p(m) = \Phi \left[\frac{k_i - \beta_i m}{\sqrt{1 - \beta_i^2}} \right]$$

Joint default, single factor

$$\pi_{12} = \Phi_2(k, k; \beta^2); \quad \rho = \frac{\Phi_2(k, k; \beta^2) - \pi^2}{\pi(1 - \pi)}$$

Φ_2 = bivariate standard normal, correlation β^2

Granular portfolio loss distribution

$$P[X \leq x] = \Phi \left[\frac{k - \sqrt{1 - \beta^2} \Phi^{-1}(x)}{\beta} \right];$$

$$\text{market factor for loss level } x: m = \frac{k - \sqrt{1 - \beta^2} \Phi^{-1}(x)}{\beta}$$

Credit VaR and granularity

Credit VaR = loss quantile - EL;

$\rho = 1$: portfolio behaves as one credit;

$\rho = 0$: number of defaults $\sim \text{Binomial}(n, \pi)$, credit VaR falls as n rises, $\rightarrow 0$ for a very granular portfolio

CR 12 Credit Risk

Value with CVA and DVA

$$f = f_{\text{nd}} - \text{CVA} + \text{DVA}; \quad \text{CVA} = \sum_i q_i v_i;$$

$$\text{DVA} = \sum_i q_i^* v_i^*$$

f_{nd} = no-default value;

q_i (q_i^*) = risk-neutral PD of counterparty (bank) in interval i ;

v_i (v_i^*) = PV of expected loss to bank (counterparty)

Interval PD from spreads

$$q_i = \exp \left[-\frac{s(t_{i-1}) t_{i-1}}{1 - R} \right] - \exp \left[-\frac{s(t_i) t_i}{1 - R} \right];$$

$$\text{survival to } t_i = \exp \left[-\frac{s(t_i) t_i}{1 - R} \right]$$

One-factor Gaussian copula PD

$$x_i = a_i F + \sqrt{1 - a_i^2} Z_i;$$

$$Q(T | F) = N \left[\frac{N^{-1}(Q(T)) - \sqrt{\rho} F}{\sqrt{1 - \rho}} \right]$$

$$a_i = \sqrt{\rho}, F = \text{common factor}$$

CR 13 Credit Derivatives

CDS spread valuation

$$\text{PV}(\text{expected payments}) + \text{PV}(\text{expected accrual}) = \text{PV}(\text{expected payoff});$$

$$\text{payments: } \sum s \cdot S(t_j) \cdot e^{-rt_j};$$

$$\text{payoff: } \sum (1 - R) \cdot \text{PD}(\text{year } j) \cdot e^{-rt_{j,\text{mid}}};$$

$$\text{accrual: } \sum \frac{1}{2} s \cdot \text{PD}(\text{year } j) \cdot e^{-rt_{j,\text{mid}}}$$

defaults mid-year, annual payments in arrears; S = survival probability

Marking a CDS to market

$$\text{Value to protection seller} = s_{\text{contract}} \times$$

$$(\text{PV payments} + \text{accrual per unit spread}) -$$

$$\text{PV}(\text{expected payoff});$$

buyer value = negative of this

Use risk-neutral PDs (implied from bond prices or CDS quotes); CDS value is insensitive to R if the same R is used to imply PD and to value.

Binary CDS spread

$$\text{Payoff} = 1 \text{ (not } 1 - R); \quad \text{binary spread} \approx \frac{\text{vanilla spread}}{1 - R}$$

Index CDS, fixed coupon price

$$P = 100 - 100 \times D \times (s - c) \text{ per } 100;$$

buyer pays $100 - P$ upfront (negative: buyer receives);

then pays coupon c on remaining notional (names not yet defaulted)

D = PV factor of spread payments, s = index spread, c = fixed coupon

Synthetic CDO breakeven spread

$$s = \frac{C}{A + B}$$

A = PV of spread payments per unit spread, B = PV of accrual payments,

C = PV of expected payoffs

CR 16 Netting and Close-out

Net notional, trade compression

Net notional = $\sum$ long notionals - $\sum$ short notionals (same reference, maturity, coupon)

net contract typically held with the counterparty of the largest original position on the same side as the net position, as a reduction of that trade

Ring (multilateral) compression

In a closed ring of fungible obligations, every leg is reduced by the smallest notional in the ring (net positions unchanged)

CR 17 Margin and Settlement

Margin, one party

$$\text{Margin} = \max(\text{value} - K_C, 0) + \text{IM}_C$$

K_C = counterparty threshold, IM_C = counterparty initial margin

Credit support amount, two-way VM

$$\text{Amount} = \max(\text{value} - K_C, 0) - \max(-\text{value} - K_P, 0) - C;$$

positive: call margin, negative: return or post;
transfer only if the amount exceeds the MTA (rounding may apply);

IM is not netted against VM

K_C, K_P = counterparty and own thresholds, C = margin already held

Haircut

$$\begin{aligned} \text{Value credited} &= (1 - x) \times \text{market value}; \\ \text{to deliver margin worth } M &\text{ post } M/(1 - x); \\ \text{haircut} &= \Phi^{-1}(\alpha) \times \sigma_C \times \sqrt{\tau} \\ \tau &= \text{liquidation time}; \end{aligned}$$

for a bond, σ_C is a yield volatility and the result is multiplied by duration

Bilateral (non-cleared)	Rule
VM	Regular (daily), zero threshold, $\text{MTA} \leq \text{€}500,000$ (same in USD), may be rehypothecated
IM	Both parties, gross (no netting), 99% confidence, 10-day horizon, segregated, no rehypothecation

CR 18 Central Clearing

Order	CCP loss waterfall
1	Defaulter IM
2	Defaulter default fund
3	CCP skin in the game
4	Non-defaulters' default fund
5	Rights of assessment and other loss allocation (VM gains haircutting, tear-up, forced allocation)
6	Remaining CCP capital
7	Liquidity support or CCP fails

CCP item	Rule
IM confidence	$\geq 99\%$
MPoR, OTC CCPs	About 5 business days
MPoR, exchange-traded	1 to 2 days
MPoR, bilateral	Minimum 10 days
Default fund	Cover 1: largest member and affiliates
Default fund, more complex risk profile or systemically important in multiple jurisdictions	Cover 2: two largest members

CR 19 Future Value and Exposure

Positive and negative exposure

$$\begin{aligned} \text{Positive exposure} &= \max(\text{value}, 0); \\ \text{negative exposure} &= \min(\text{value}, 0); \\ \text{with margin: } &\max(\text{value} - \text{margin}, 0), \\ &\min(\text{value} - \text{margin}, 0) \end{aligned}$$

Metric (scenario set)	Definition
EFV	Average of all values
EPE (EE)	Average with negative values set to 0
ENE (NEE)	Average with positive values set to 0
PFE	Exposure at a high confidence level
Maximum PFE	Highest PFE over the profile
Average EPE	Time-weighted average of EPE (loan equivalent)

Square-root-of-time profiles

Single final payment (FX forward): exposure $\propto \sqrt{t}$;

swap (roll-off): exposure $\propto (T - t)\sqrt{t}$, maximum at $t = T/3$

Margin, exposure and funding

Positive exposure = $\max(\text{value} - \text{VM} - \text{IM}^R, 0)$;

negative exposure = $\min(\text{value} - \text{VM} + \text{IM}^P, 0)$;

funding = $\text{value} - \text{VM} + \text{IM}^P$

IM^R = IM received, IM^P = IM posted

MPoR	Days
Basel II minimum (OTC, daily calls)	10 business days; add contractual days between calls
Basel III, certain cases	20 days
CCPs	About 5 days

Effective EE and effective EPE

Effective $\text{EE}_k = \max(\text{Effective EE}_{k-1}, \text{EE}_k)$;

Effective EPE = time-weighted average of effective EE over the first year

Netting factor

$$\text{Netting factor} = \frac{\sqrt{n + n(n-1)\bar{\rho}}}{n};$$

$\bar{\rho} = 0$: $1/\sqrt{n}$ (two exposures 71%, four 50%); $\bar{\rho} = 1$: 100%;

$$\bar{\rho} \geq -1/(n-1)$$

$\bar{\rho}$ = average correlation

CR 20 CVA

Unilateral CVA

$$\text{UCVA}(t) = -\text{LGD} \sum_{i=1}^m \text{EPE}(t, t_i) \times \text{PD}(t_{i-1}, t_i)$$

EPE discounted; PD = marginal default probability in each interval;
CVA negative (a cost) in this chapter

CVA as a running spread

$$\text{UCVA} \approx -\text{average EPE} \times \text{spread};$$

$$\text{running spread} = \frac{\text{upfront CVA}}{\text{risky annuity}}$$

per annum, same units as the credit spread;
risky annuity = risky duration $\times$ notional

CVA, actual vs market LGD

$$\text{UCVA}(t) = -\text{LGD}_{\text{actual}} \sum \text{EPE}(t, t_i) \times [e^{-s_{i-1}t_{i-1}/\text{LGD}_{\text{mkt}}} - e^{-s_i t_i/\text{LGD}_{\text{mkt}}}]$$

If $\text{LGD}_{\text{actual}} = \text{LGD}_{\text{mkt}}$ the LGD terms cancel to first order (higher recovery raises implied PD but cuts LGD, so CVA barely moves), and CVA generally rises in magnitude as spreads widen.

CVA and DVA with survival

$$CVA = -LGD_C \sum EPE(t, t_i) \times PD_C(t_{i-1}, t_i) \times [1 - PD_P(0, t_{i-1})];$$

$$DVA = -LGD_P \sum ENE(t, t_i) \times PD_P(t_{i-1}, t_i) \times [1 - PD_C(0, t_{i-1})];$$

$$BCVA = CVA + DVA$$

P = party calculating, C = counterparty;

DVA positive since ENE negative

BCVA as a spread

$$BCVA = -\text{average EPE} \times \text{Spread}_C - \text{average ENE} \times \text{Spread}_P;$$

if average EPE = -average ENE:

$$BCVA = -EPE \times (\text{Spread}_C - \text{Spread}_P)$$

Incremental and marginal CVA

$$CVA^{NS} \geq \sum_i CVA_i \text{ (netted CVA never more negative than the sum of standalone);}$$

$$CVA^{NS \rightarrow NS^*} = CVA^{NS^*} - CVA^{NS} =$$

$$-LGD \sum EPE^{NS \rightarrow NS^*}(t, t_i) \times PD(t_{i-1}, t_i);$$

marginal CVA: same with marginal EPE, contributions sum to total CVA

NS = netting set, NS^* = netting set with the new trade

OC diversion and equity flow

$$OC_t = \min(L_t - B, K) \text{ if } L_t \geq B;$$

$$OC_t = \max\{L_t - B, -[(1+r)OC_{\text{account}_{t-1}} + R_t]\} \text{ if } L_t < B;$$

$$\text{equity flow} = \max(L_t - B - OC_t, 0)$$

K = cap on diversion

Recovery and OC account

$$R_t = 0.4 \times d_t \times \text{loan par};$$

$$OC_{\text{account}_t} = OC_t + R_t + (1+r) \times OC_{\text{account}_{t-1}}$$

Terminal funds and tranche losses

$$F = (1+r)OC_{\text{account}_{T-1}} +$$

$$[(N - \sum d)(1 + c_{\text{loan}}) + 0.4d_T] \times \text{loan par};$$

$$\text{senior loss} = \max[P_S(1 + c_S) - F, 0];$$

$$\text{mezzanine loss} = \max[P_M(1 + c_M) - (F - P_S(1 + c_S)), 0];$$

$$\text{equity} = \max[F - P_S(1 + c_S) - P_M(1 + c_M), 0]$$

P, c = tranche par and coupon (S senior, M mezzanine);

r = OC account rate; c_{loan} = loan coupon

default01

$$\text{default01} = \frac{1}{20} \times [V(\pi + 0.0010) - V(\pi - 0.0010)]$$

V = mean value or loss; quoted positive, per 1 bp of default probability

CR 21 Stress Testing Counterparty Exposures

Loan portfolio stress loss

$$EL = \sum_i p_i \cdot ead_i \cdot lgd_i; \quad EL_s = \sum_i p_i^s \cdot ead_i \cdot lgd_i;$$

$$\text{stress loss} = EL_s - EL$$

Derivative portfolio stress loss

$$EL = \sum_i p_i \cdot \alpha \cdot epe_i \cdot lgd_i; \quad EL_s = \sum_i p_i^s \cdot \alpha \cdot epe_i^s \cdot lgd_i;$$

$$\text{stress loss} = EL_s - EL$$

stress PD, EPE or both

Stressed CVA and stress loss

$$CVA_n = LGD_n^* \sum_j EE_n^*(t_j) \cdot q_n^*(t_{j-1}, t_j);$$

$$CVA = \sum_n CVA_n; \quad CVA_s \text{ uses stressed LGD, EE, } q;$$

$$\text{stress loss} = CVA_s - CVA$$

EE^* discounted; q^* = risk-neutral marginal PD

Bilateral CVA with DVA

$$BCVA = \sum_n LGD_n^* \sum_j EE_n^*(t_j) \cdot q_n^*(t_{j-1}, t_j) \cdot S_I^*(t_{j-1}) - \sum_n LGD_I^* \sum_j NEE_n^*(t_j) \cdot q_I^*(t_{j-1}, t_j) \cdot S_n^*(t_{j-1});$$

DVA = the second sum; stress loss = stressed BCVA - BCVA

I = the institution, S = survival probability

CR 21 writes CVA as a positive cost and $BCVA = CVA - DVA$; CR 20 writes CVA as negative and $BCVA = CVA + DVA$.

CR 22 Structured Credit Risk

Tranche loss

$$\text{Tranche writedown} = \frac{\min[\max(\text{pool loss} - A, 0), D - A]}{\text{tranche size}}$$

A = attachment, D = detachment

Collateral interest

$$L_t = (r + \text{spread}) \times (N - \sum_{\tau \leq t} d_\tau) \times \text{loan par};$$

$$\text{excess spread} = L_t - B$$

N = number of loans, d = defaults, B = bond coupons due

CR 23 Introduction to Securitization

Delinquency and default ratios

$$\text{Delinquency ratio} = \frac{\text{receivables overdue } > 90 \text{ days}}{\text{total receivables}};$$

$$\text{default ratio} = \frac{\text{receivables written off in period}}{\text{total receivables at period end}} \text{ credit cards}$$

Monthly payment rate

$$\text{MPR} = \frac{\text{principal and interest repaid in period}}{\text{pool balance}}$$

collections / outstanding pool balance; not a prepayment measure

DSCR

$$\text{DSCR} = \frac{\text{net operating income}}{\text{debt payments}};$$

below 1.0: cash flow does not cover debt payments

WAC and WAM

$$\text{WAC} = \frac{\sum(\text{loan rate} \times \text{loan balance})}{\sum \text{loan balance}};$$

$$\text{WAM} = \frac{\sum(\text{remaining term in months} \times \text{loan balance})}{\sum \text{loan balance}}$$

WAL

$$\text{WAL} = \sum \frac{a}{365} \times \text{PF}(t)$$

a = actual days in each payment period (period length, not elapsed time),
PF = pool factor

SMM, CPR, PSA

$$\text{SMM} = \frac{\text{prepayment in month}}{\text{start-of-month balance} - \text{scheduled principal}};$$

$$\text{CPR} = 1 - (1 - \text{SMM})^{12}; \quad \text{PSA} = \frac{\text{CPR}}{0.2 \times m} \times 100;$$

100% PSA: CPR rises 0.2% a month to 6% at month 30, then flat

m = months since origination

OPERATIONAL RISK AND RESILIENCE

OR 4 Risk Measurement and Assessment

Fault tree, AND conditions

$$P(\text{scenario}) = P_1 \times P_2 \times \dots \times P_n;$$

e.g. three controls each failing 10%: $10\%^3 = 1/1,000$
independent conditions; the theoretical minimum likelihood

Fault tree, OR conditions

$$P(\text{at least one}) = 1 - (1 - P_1)(1 - P_2) \dots (1 - P_n)$$

independent conditions

Scaling by unit of exposure

Expected occurrences = probability per unit $\times$ number of units of exposure;

e.g. $1/1,000,000$ per employee $\times$ 100,000 employees = 10%

Heatmap ratings are ordinal, so never multiply likelihood by impact: 1×4 (frequent, low impact) is not the same risk as 4×1 (remote, extreme impact).

OR 18 Risk Capital Attribution and RAROC

RAROC

$$\text{RAROC} = \frac{\text{After-tax expected risk-adjusted net income}}{\text{Economic capital}};$$

capital budgeting form: $\text{RAROC} = \frac{\text{Expected revenues} - \text{Costs} - \text{EL} - \text{Taxes} + \text{Return on risk capital} \pm \text{Transfers}}{\text{Economic capital}}$

RAROC inputs

$$\text{Return on risk capital} = \text{risk capital} \times r_f;$$

$$\text{after-tax income} = \text{pre-tax} \times (1 - t);$$

ex post performance: realized revenues and losses in place of expected
 t = effective tax rate

Economic capital

$$\text{EC} = \text{risk capital} + \text{strategic risk capital};$$

$$\text{strategic risk capital} = \text{goodwill} + \text{burned-out capital};$$

$$\text{risk capital} = \text{worst-case loss at the confidence level} - \text{EL (unexpected loss)}$$

Confidence level from target rating

$$\text{Confidence level} = 1 - \text{target-rating one-year PD};$$

$$\text{AAA } 0.01\% \rightarrow 99.99\%;$$

$$\text{AA } 0.03\% \rightarrow 99.97\% \text{ (AA 3 to 5 bp} \rightarrow 99.95 \text{ to } 99.97\%)$$

Hurdle rate

$$h_{AT} = \frac{CE \times r_{CE} + PE \times r_{PE}}{CE + PE};$$

$$r_{CE} = r_f + \beta_{CE}(R_M - r_f); \text{ accept if } \text{RAROC} > h_{AT}$$

CE, PE = market values of common and preferred equity;

r_{PE} = preferred yield

Adjusted RAROC

$$\text{Adjusted RAROC} = \text{RAROC} - \beta_E(R_M - r_f);$$

$$\text{accept if adjusted RAROC} > r_f$$

β_E = beta of the firm's equity

Market risk capital over one year

$$\text{Naive: annualized VaR} = \text{daily VaR} \times \sqrt{252};$$

$$\text{RC} = \sqrt{\sum \text{VaR}_i^2 + \text{VaR}_{\text{core}}^2} \times \text{remaining days}$$

with a core risk level; sum over the time-to-reduce days; days total 252

Aggregate risk capital bounds

Perfect correlation: $\sum \text{RC}_i$; zero correlation: $\sqrt{\sum \text{RC}_i^2}$;
firm capital lies between the two

Allocating capital in a business unit

$$\text{Diversification effect} = \sum \text{stand-alone RC} - \text{RC}_{BU};$$

$$\text{fully diversified } \text{RC}_i = \text{stand-alone } \text{RC}_i - \text{effect} \times \frac{\text{RC}_i}{\sum \text{RC}}$$

(pro rata);

$$\text{marginal } \text{RC}_i = \text{RC}_{BU} - \text{RC}_{BU \text{ without } i};$$

$$\sum \text{marginal} < \text{RC}_{BU}$$

OR 21 Capital Regulation Before the GFC

Basel I capital ratios

$$\text{Tier 1/RWA} > 4\%;$$

$$\text{total capital/RWA} > 8\% \text{ (Cooke ratio);}$$

$$\text{total capital} = \text{Tier 1} + \text{Tier 2}; \text{ Tier 2} \leq \text{half of total capital};$$

$$\text{loan loss reserves in capital} \leq 1.25\% \text{ of RWA (originally } 2\%)$$

Basel I RWA

$$\text{RWA} = \sum w_i A_i; \text{ capital} = 8\% \times \text{RWA};$$

off-balance sheet: credit equivalent = CCF $\times$ principal, then $\times$ counterparty risk weight;
derivative risk weight capped at 50%

Weight	Basel I on-balance-sheet exposures
0%	Cash, claims on OECD governments, full OECD government guarantee
20%	OECD banks and OECD public sector entities (municipalities, Fannie Mae, Freddie Mac)
50%	Uninsured residential mortgages
100%	All other (corporate, consumer loans)

CCF	Basel I off-balance-sheet items
100%	Guarantees on loans and bonds, bankers acceptances
50%	Warranties, transaction-related standby letters of credit
20%	Loan commitments, original maturity ≥ 1 year
0%	Loan commitments < 1 year

Derivatives, current exposure method

$$\text{Credit equivalent} = \max(V, 0) + \text{add-on factor} \times L$$

L = notional; add-on factors in % of notional below

Maturity	Interest rate add-on	FX add-on
< 1 yr	0%	1%
1 to 5 yr	0.5%	5%
> 5 yr	1.5%	7.5%

Netting (1995 amendment)

$$\text{CEA} = \sum_c \max(\sum_{i \in c} V_i, 0) + \sum_j (0.4 D_j + 0.6 \text{NRR } D_j);$$

$$\text{NRR} = \frac{\sum_c \max(\sum_{i \in c} V_i, 0)}{\sum_i \max(V_i, 0)} = \frac{\text{net current exposure}}{\text{gross positive exposure}}$$

netting only within each counterparty c ;

D_j = add-on factor $\times$ total notional of derivative type j (a currency amount)

1996 market risk charge (internal models)

$$MR = \max(VaR_{t-1}, m \times VaR_{avg});$$

10-day 99% VaR ($\sqrt{10} \times$ one-day allowed), VaR_{avg} over 60 days, $m \geq 3$;

specific risk multiplier 4;

$$\text{market RWA} = 12.5 \times MR \quad (12.5 = 1/8\%)$$

Backtesting multiplier

1-day 99% VaR vs actual loss over the last 250 days;

$m = 3$ for fewer than 5 exceptions;

5, 6, 7, 8, 9 exceptions: 3.40, 3.50, 3.65, 3.75, 3.85;
10 or more: 4

same zones as the MR 4 backtesting table

IRB capital

$$\text{Capital} = \sum [EAD_i \times LGD_i \times DR_{99.9,i}] - EL;$$

$$EL = \sum EAD_i \times LGD_i \times PD_i;$$

$$DR_{99.9,i} = N \left[\frac{N^{-1}(PD_i) + \sqrt{\rho} N^{-1}(0.999)}{\sqrt{1-\rho}} \right]$$

IRB RWA, bank, corporate, sovereign

$$RWA = 12.5 \times EAD \times LGD \times (DR - PD) \times MA;$$

$$MA = \frac{1 + (M - 2.5)b}{1 - 1.5b}; \quad b = [0.11852 - 0.05478 \ln(PD)]^2;$$

F-IRB: $PD \geq 0.03\%$ (bank, corporate), LGD 45% senior, 75% subordinated, $M = 2.5$ in most cases

M = effective maturity; MA = maturity adjustment

IRB correlation

$$\text{Bank, corporate, sovereign: } \rho = 0.12 \frac{1 - e^{-50PD}}{1 - e^{-50}} + 0.24 \left[1 - \frac{1 - e^{-50PD}}{1 - e^{-50}} \right];$$

retail: residential mortgages $\rho = 0.15$, qualifying revolving $\rho = 0.04$, other retail same form with 0.03, 0.16 and $-35PD$;
retail: no MA

Basic indicator approach

Capital = 15% $\times$ average annual gross income over the past three years, ignoring years of negative gross income

Standardized approach (operational)

Each year: capital = $\sum \beta_i \times GI_i$ over 8 business lines (negatives offset within the year);

requirement = average over three years, ignoring years with negative total

β	Business lines
18%	Corporate finance, trading and sales, payment and settlement
15%	Commercial banking, agency services
12%	Retail banking, asset management, retail brokerage

A year with a negative total is excluded from the BIA or TSA average (divide by the number of positive years), not floored at zero and kept.

Standard	Confidence	Horizon
Market risk (1996)	99%	10-day
IRB credit	99.9%	1 year
AMA operational (insurance offset $\leq 20\%$)	99.9%	1 year
Solvency II SCR	99.5%	1 year

OR 22 Post-Crisis Solvency and Liquidity Regulation

Basel 2.5 market risk charge

$$MR_{2.5} = \max(VaR_{t-1}, m_c VaR_{avg}) + \max(SVaR_{t-1}, m_s SVaR_{avg});$$

10-day 99%, averages over 60 days, $m_c, m_s \geq 3$

$SVaR$ from the most stressful one-year (250-day) period in the past 7 years

With equal multipliers, $MR_{2.5} \geq 2 \times MR_{1996}$: stressed VaR is added to VaR, it does not replace it.

Incremental risk charge

99.9th percentile, one-year horizon, constant level of risk (downgraded or defaulted positions replaced);
liquidity horizon never less than 3 months

Basel III minimum ratios

$$CET1/RWA \geq 4.5\%;$$

$$\text{Tier 1 (CET1+AT1)/RWA} \geq 6\%;$$

$$\text{total capital (Tier 1+Tier 2)/RWA} \geq 8\%;$$

general reserves in Tier 2 $\leq 1.25\%$ of standardized RWA or 0.6% of IRB RWA

Basel III capital components

Core Tier 1 = common equity + retained earnings + limited minority interest and unrealized gains and losses – goodwill and other intangibles – deferred tax assets – shortfall of reserves vs IRB expected losses;

Additional Tier 1 = non-cumulative perpetual preferred (callable only after ≥ 5 years) + debt with conversion or write-down triggers + minority interest not in Core Tier 1;
Tier 2 = subordinated debt (≥ 5 years original maturity, callable only after ≥ 5 years) + general loan loss reserves (cap above);
other deductions: defined-benefit pension deficits, certain cross-holdings, mortgage servicing rights $> 10\%$ of common equity

Basel III buffers

$$CCB = 2.5\% \text{ of RWA};$$

CCyB = 0 to 2.5% of RWA (international bank: weighted average of each jurisdiction's CCyB);

G-SIB surcharge = 1, 1.5, 2, 2.5 or 3.5%;

CET1 requirement

$$= 4.5\% + 2.5\% + \text{G-SIB surcharge} (+ \text{CCyB})$$

Leverage ratio

$$\text{Leverage ratio} = \frac{\text{Tier 1 capital}}{\text{Exposure measure}} \geq 3\%;$$

exposure measure = on-balance-sheet assets + fractions of off-balance-sheet exposures
the OR 22 text uses core Tier 1

Liquidity coverage ratio

$$LCR = \frac{HQLA}{\text{Net cash outflows over 30 days}} \geq 100\%;$$

$$HQLA = \sum \text{assets} \times (1 - \text{haircut});$$

Level 2 assets $\leq 40\%$ of HQLA after haircut

Net stable funding ratio

$$NSFR = \frac{\text{Available stable funding}}{\text{Required stable funding}} \geq 100\%;$$

$$ASF = \sum \text{funding} \times \text{ASF factor};$$

$$RSF = \sum \text{assets} \times \text{RSF factor}$$

one-year horizon

CoCo capital classification

Trigger $CET1/RWA \geq 5.125\%$: Additional Tier 1;
lower trigger: Tier 2

OR 23 Basel III Reforms Summary

Output floor

$$RWA = \max(RWA_{IM}, 72.5\% \times RWA_{SA})$$

RWA_{IM} = total RWA using approved approaches, including internal models;

RWA_{SA} = total RWA using only the standardized approaches

G-SIB leverage ratio buffer

Buffer = 50% × G-SIB risk-weighted higher-loss-absorbency requirement, met with Tier 1;

Tier 1 leverage requirement = 3% + buffer

OR 24 Basel III Operational Risk Capital

Business indicator

$$BI = ILDC + SC + FC;$$

$$ILDC = \min[|II - IE|, 2.25\% \times IEA] + DI;$$

$$SC = \max(OOI, OOE) + \max(FI, FE);$$

$$FC = |\text{Net P\&L trading book}| + |\text{Net P\&L banking book}|$$

II, IE = interest income, expense; IEA = interest earning assets;

DI = dividend income; OOI, OOE = other operating income, expense;

FI, FE = fee income, expense;

each term a three-year average ($t, t-1, t-2$), absolute values taken year by year before averaging

Business indicator component

$$BIC = \sum \alpha_i BI_i \text{ (marginal); bucket 1, } BI \leq \text{€1bn: } 12\%;$$

$$\text{bucket 2, } \text{€1bn} < BI \leq \text{€30bn: } 15\%;$$

$$\text{bucket 3, } BI > \text{€30bn: } 18\%$$

Each coefficient applies only to the slice of BI inside its bucket (like tax slabs), never to the whole BI.

Loss component and ILM

$LC = 15 \times$ average annual operational risk losses over the previous 10 years;

$$ILM = \ln(e^1 - 1 + (LC/BIC)^{0.8});$$

$$LC = BIC \Rightarrow ILM = 1; LC > BIC \Rightarrow ILM > 1$$

Operational risk capital

$$ORC = BIC \times ILM; RWA_{op} = 12.5 \times ORC;$$

$$\text{bucket 1: } ILM = 1, ORC = 12\% \times BI;$$

supervisor may set $ILM = 1$ for all banks;

loss data standards not met: capital $\geq 100\%$ of BIC;

fewer than 5 years of data: BIC only

Loss data rules

10-year observation period (5 years acceptable on first moving to the SA);

loss inclusion threshold €20,000, raisable to €100,000 for buckets 2 and 3;

losses net of recoveries, including insurance (receivables and tax effects are not recoveries)

LIQUIDITY AND TREASURY RISK MEASUREMENT AND MANAGEMENT

LR 1 Liquidity Risk

Bid-offer spread

$$p = \text{Offer price} - \text{Bid price}; s = \frac{\text{Offer price} - \text{Bid price}}{\text{Mid-market price}}$$

Cost of liquidation, normal market

$$\sum_i \frac{s_i \alpha_i}{2}$$

s_i = proportional spread, α_i = dollar mid-market value of position i

Cost of liquidation, stressed market

$$\sum_i \frac{(\mu_i + \lambda \sigma_i) \alpha_i}{2}$$

μ_i, σ_i = mean and SD of the proportional spread;

$\lambda = 2.326$ at 99% if spreads are normal

Liquidity-adjusted VaR

$$LVaR = VaR + \sum_i \frac{s_i \alpha_i}{2};$$

$$\text{stressed: } LVaR = VaR + \sum_i \frac{(\mu_i + \lambda \sigma_i) \alpha_i}{2}$$

Basel III LCR and NSFR

$$LCR = \frac{\text{High-quality liquid assets}}{\text{Net cash outflows in 30 days}} \geq 100\%;$$

$$NSFR = \frac{\text{Amount of stable funding}}{\text{Required amount of stable funding}} \geq 100\%$$

NSFR: funding × ASF factor over assets and off-balance-sheet items × RSF factor

LR 2 Liquidity and Leverage

Leverage ratio

$$L = \frac{A}{E} = \frac{E + D}{E} = 1 + \frac{D}{E};$$

$$\text{collateralized position with haircut } h: L = \frac{1}{h}$$

Leverage effect

$$r^e = Lr^a - (L-1)r^d; \frac{\partial r^e}{\partial L} = r^a - r^d$$

r^a = return on assets, r^d = cost of debt, r^e = return on equity

Levered return on repo-financed bond

$$\frac{c - (1-h)r}{h} = c + \frac{1-h}{h}(c-r)$$

c = coupon, h = haircut, r = repo rate

Expected transactions cost

$$E[P_{t+1}] \times \frac{\bar{s}}{2}; s = \frac{2(\text{ask} - \text{bid})}{\text{ask} + \text{bid}} = \frac{\text{ask} - \text{bid}}{\text{midprice}}$$

Spread risk factor (99%)

$$\frac{1}{2}(\bar{s} + 2.33 \sigma_s);$$

$$99\% \text{ CI on transactions cost per unit: } \pm \bar{P} \times \frac{1}{2}(\bar{s} + 2.33 \sigma_s)$$

LVaR for liquidation over T days

$$VaR \times \sqrt{\frac{(1+T)(1+2T)}{6T}}; T = 5: \times 1.4832;$$

$$T = 10: \text{about } \times 2$$

equal daily sales; VaR = one-day position VaR

Malz scales VaR by $\sqrt{(1+T)(1+2T)/(6T)}$, not by $\sqrt{T}$, while Hull adds the spread cost to VaR .

LR 4 The Investment Function

After-tax yield and TEY

After-tax gross yield = Before-tax gross yield $\times (1 - t)$;

$$\text{TEY} = \frac{\text{After-tax return on tax-exempt investment}}{1 - t}$$

t = marginal income tax rate

Duration price change and immunization

$$\% \Delta P \approx -D \frac{\Delta i}{1 + i};$$

$$m \text{ payments a year: } \% \Delta P \approx -D \frac{\Delta i}{1 + i/m};$$

immunize: D = planned holding period

Expected liquidity

$$\text{TSL}_e(t_0, t_i) = \text{TSECCF}(t_0, t_i) + \text{TSCCLGC}(t_0, t_i);$$

$$\text{TSECCF}(t_0, t_0) = \text{Cash}(t_0);$$

solvency requires $\text{TSL}_e \geq 0$ at every date

Cash flow at risk

$$\text{cfaR}_\alpha(t_0, t_i) = \text{cf}_\alpha(t_0, t_i; x) - \text{cf}_e(t_0, t_i; x);$$

$$\text{cfaR}_{1-\alpha}(t_0, t_i) = \text{cf}_{1-\alpha}(t_0, t_i; x) - \text{cf}_e(t_0, t_i; x)$$

cf = total net cash flow on t_i ; $\text{cf}_e = E[\text{cf}]$; x = vector of risk factors

Repo cash and repurchase amount

$$\text{Cash received} = N \times (\text{Clean price} + \text{Accrued}) \times (1 - h);$$

$$\text{Repayment} = \text{Cash received} \times (1 + r_{\text{repo}} \tau)$$

N = notional, h = haircut, τ = repo term in years

LR 5 Liquidity and Reserves Management

Net liquidity position

L = (Incoming deposits + Revenues from nondeposit services + Customer loan repayments + Sales of assets + Borrowings from money market) – (Deposit withdrawals + Volume of acceptable loan requests + Repayments of borrowings + Other operating expenses + Dividend payments)

$L < 0$: deficit; $L > 0$: surplus

Sources and uses of funds

Estimated liquidity deficit (–) or surplus (+) = Estimated change in deposits – Estimated change in loans

Total liquidity requirement (structure of funds)

$$\text{Total} = 0.95 (\text{Hot money funds} - \text{RR})$$

$$+ 0.30 (\text{Vulnerable funds} - \text{RR})$$

$$+ 0.15 (\text{Stable funds} - \text{RR})$$

$$+ 1.00 (\text{Potential loans} - \text{Actual loans})$$

RR = required legal reserves held against that category

Expected liquidity requirement

$$\sum P(\text{outcome}) \times \text{estimated liquidity surplus or deficit in that outcome}$$

Liquidity indicator	Ratio
Cash position	Cash and due from depository institutions / Total assets
Liquid securities	U.S. government securities / Total assets
Net fed funds and repo position	(Fed funds sold + reverse repos – Fed funds purchased – repos) / Total assets
Capacity (negative)	Net loans and leases / Total assets
Pledged securities (negative)	Pledged securities / Total securities
Hot money	Money market assets / Volatile liabilities
Deposit brokerage	Brokered deposits / Total deposits
Core deposit	Core deposits / Total assets
Deposit composition	Demand deposits / Time deposits
Loan commitments	Unused loan commitments / Total assets

LR 7 Monitoring Liquidity

Cumulated expected cash flow

$$\text{CF}(t_0, t_a, t_b) = \sum_{i=a}^b [\text{cf}_e^+(t_0, t_i) + \text{cf}_e^-(t_0, t_i)];$$

$$\text{TSECCF} = \{\text{CF}(t_0, t_0, t_1), \dots, \text{CF}(t_0, t_0, t_b)\}$$

LR 9 Liquidity Stress Testing

Stressed liquid asset buffer

Normal liquid asset buffer – Stressed cash outflows + Stressed cash inflows

LR 12 Managing Non-Deposit Liabilities

Available funds gap

AFG = Current and projected loans and investments the institution desires to make – Current and expected deposit inflows and other available funds

Effective cost of a funding source

$$\text{Effective cost rate} = \frac{\text{Current interest cost} + \text{Noninterest costs}}{\text{Net investable funds}};$$

Interest cost = Money market rate $\times$ Amount borrowed;

Noninterest cost = Cost rate $\times$ Amount borrowed;

Net investable funds = Amount borrowed – Legal reserves – Deposit insurance – Funds placed in nonearning assets

Historical average cost of funds

$$\text{Weighted average interest expense} = \frac{\text{All interest paid}}{\text{Total funds raised}};$$

$$\text{Break-even cost rate} = \frac{\text{Interest} + \text{Other operating costs}}{\text{All earning assets}};$$

$$\text{Weighted average overall cost of capital} = \text{Break-even cost} + \left[\frac{\text{After-tax cost of equity}}{1 - t} \right] \times \frac{\text{Stockholders' investment}}{\text{Earning assets}}$$

Pooled-funds cost and hurdle rate

$$\text{Pooled deposit and nondeposit funds expense} = \frac{\text{All expected operating expenses}}{\text{All new funds expected}};$$

$$\text{Hurdle rate over all earning assets} = \frac{\text{All expected operating costs}}{\text{Dollars available to place in earning assets}}$$

LR 13 Repurchase Agreements and Financing

Repo repurchase price

$$\text{Repurchase price} = \text{Invoice price (incl. accrued)} \times \left(1 + \text{repo rate} \times \frac{\text{days}}{360} \right)$$

actual/360

Special spread and fails penalty

Special spread = GC rate – Special rate;

Fails penalty rate = $\max(3\% - \text{fed funds target}, 0)$

upper limit of the special spread: GC rate before May 2009, penalty rate after

Financing advantage of a special

$$\text{Financing value} = \text{Market value} \times \text{Special spread} \times \frac{\text{days}}{360}$$

e.g. $100 \times 125 \times 0.22\% / 360 = 0.076$ per 100 market value

LR 14 Liquidity Transfer Pricing

Cost of contingent liquidity risk

$$\text{Charge rate} = \frac{\text{Limit} - \text{Drawn amount}}{\text{Limit}} \times \text{Likelihood of drawdown} \times \text{Cost of funding liquidity cushion};$$

$$\text{Dollar charge} = \text{Charge rate} \times \text{Limit}$$

e.g. $(10 - 4) / 10 \times 0.6 \times 0.0018 = 6.48$ bps; $\times \$10m = \$6,480$

Funds transfer price

$$\text{FTP} = \text{Base rate} + \text{Term liquidity premium} + \text{Liquidity premium}$$

liquidity premium = cost of carrying the liquidity cushion / total assets,
e.g. $\$30m / \$300bn = 1$ bp

LR 16 Covered Interest Parity Lost

Covered interest parity

$$\frac{F}{S} = \frac{1+r}{1+r^*};$$

$$\text{USD at a premium in FX swaps: } F - S > S \left[\frac{1+r}{1+r^*} - 1 \right]$$

S, F in USD per unit of foreign currency;
 r = USD rate, r^* = foreign rate

Cross-currency basis

$$F - S = S \frac{1+r+b}{1+r^*} - S;$$

with r^* small: $b = \frac{F-S}{S} - (r - r^*);$

drop in market liquidity: $b = \frac{F}{S} - \frac{F_b}{S_a};$

credit or other risk premia: $b = rp;$

hedging demand with balance sheet costs: $b \propto rp \times \text{FX hedging demand}$

S_a = spot ask, F_b = forward bid

LR 17 Asset-Liability Management

Net interest margin

$$\text{NIM} = (\text{Interest income from loans and investments} - \text{Interest expense on deposits and other borrowed funds}) / \text{Total earning assets}$$

Interest-sensitive gap measures

$$\text{ISGAP} = \text{ISA} - \text{ISL}; \quad \text{Relative ISGAP} = \frac{\text{ISGAP}}{\text{Total assets}};$$

$$\text{Interest sensitivity ratio} = \frac{\text{ISA}}{\text{ISL}}$$

Change in net interest income

$$\Delta \text{NII} = \Delta i \times \text{Cumulative gap}$$

Δi in percentage points, gap in dollars

Duration

$$D = \frac{\sum_t CF_t t / (1 + \text{YTM})^t}{\sum_t CF_t / (1 + \text{YTM})^t} = \frac{\sum_t CF_t t / (1 + \text{YTM})^t}{\text{Price}};$$

$$D_{\text{portfolio}} = \sum (\text{Market value weight} \times D_i)$$

Leverage-adjusted duration gap

$$\text{Duration gap} = D_A - D_L \times \frac{\text{Total liabilities}}{\text{Total assets}}$$

Change in net worth

$$\Delta \text{NW} = \left[-D_A \frac{\Delta i}{1+i} A \right] - \left[-D_L \frac{\Delta i}{1+i} L \right];$$

$$\approx - \left[D_A - \frac{L}{A} D_L \right] \times A \times \frac{\Delta i}{1+i}$$

i = original rate

Gap	Rates up	Rates down
IS gap > 0 (asset sensitive; relative ISGAP > 0, ISR > 1)	NIM up	NIM down
IS gap < 0 (liability sensitive)	NIM down	NIM up
Duration gap > 0	NW down	NW up
Duration gap < 0	NW up	NW down
Zero gap	Immunized	Immunized

The duration gap scales D_L by L/A , so a zero gap needs $D_L = D_A \times A/L$ (slightly above D_A).

RISK MANAGEMENT AND INVESTMENT MANAGEMENT

IM 1 Factor Theory

CAPM: market risk premium and SML

$$E(r_m) - r_f = \bar{\gamma} \sigma_m^2;$$

$$E(r_i) - r_f = \beta_i [E(r_m) - r_f], \quad \beta_i = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)}$$

$\bar{\gamma}$ = risk aversion of the average investor

SDF pricing

$$P_i = E[m \times \text{payoff}_i]; \quad 1 = E[m(1 + r_i)];$$

$$\text{risk-free: } \frac{1}{1 + r_f} = E[m]$$

m = stochastic discount factor

SDF risk premium (beta form)

$$E(r_i) - r_f = \left[\frac{\text{cov}(r_i, m)}{\text{var}(m)} \right] \times \left[\frac{-\text{var}(m)}{E(m)} \right] = \beta_{i,m} \times \lambda_m$$

payoff high in bad times (high cov with m) earns a lower premium

IM 2 Factors

Fama-French model (plus momentum)

$$E(r_i) = r_f + \beta_{i,\text{MKT}} E(r_m - r_f) + \beta_{i,\text{SMB}} E(\text{SMB}) + \beta_{i,\text{HML}} E(\text{HML});$$

momentum version adds $+ \beta_{i,\text{WML}} E(\text{WML})$

IM 3 Alpha and the Low-Risk Anomaly

Alpha, tracking error, IR

$$r_t^{ex} = r_t - r_t^{bmk}; \quad \alpha = \frac{1}{T} \sum r_t^{ex}; \quad \text{TE} = \bar{\sigma} = \text{stdev}(r_t^{ex});$$

$$\text{IR} = \frac{\alpha}{\bar{\sigma}}$$

Sharpe ratio (benchmark = r_f)

$$\alpha = \text{mean}(r_t - r_{ft}); \quad \text{SR} = \frac{\text{mean}(r_t - r_{ft})}{\bar{\sigma}}$$

IR with a risk-free benchmark = SR

Fundamental law of active management

$$IR \approx IC \times \sqrt{BR}$$

IC = information coefficient;

BR = number of independent bets per year

CAPM benchmark (mimicking portfolio)

$$E(r_i) = (1 - \beta) r_f + \beta E(r_m); \quad r_t^{bmk} = (1 - \beta) r_t^f + \beta r_t^m;$$

$$\alpha = \text{mean}(r_t - r_t^{bmk})$$

Factor regression (CAPM, Fama-French, momentum)

$$r_{it} - r_{ft} = \alpha + \beta(r_{mt} - r_{ft}) +$$

$$s \text{ SMB}_t + h \text{ HML}_t + u \text{ UMD}_t + \varepsilon_{it};$$

benchmark = (1 - β) in T-bills + β in market

$$+ s(\text{small} - \text{large}) + h(\text{value} - \text{growth})$$

IM 4 Portfolio Construction

Alpha structure and scale

$$\alpha = \text{volatility} \times IC \times \text{score}; \quad \text{Std}\{\alpha\} \approx \text{volatility} \times IC$$

score has mean 0 and SD 1

Trim outliers: examine alphas with magnitude above 3 times the scale, and pull genuine ones back to plus or minus 3 times the scale.

Benchmark neutralization

$$\alpha_n(\text{neutral}) = \alpha_n - \beta_n \times \alpha_B$$

Optimal active risk aversion

$$\lambda_A = \frac{IR}{2\psi_P}$$

ψ_P = active risk, in percent (not decimals)

Annualized transaction cost

$$\text{annualized TC} = \frac{\text{round-trip cost}}{\text{holding period (years)}}$$

Marginal contribution to value added

$$\text{MCVA}_n = \alpha_n - 2\lambda_A \times \psi \times \text{MCAR}_n$$

MCAR = marginal contribution to active risk

No-trade region

$$-SC_n \leq \text{MCVA}_n \leq PC_n;$$

$$2\lambda_A \psi \text{MCAR}_n - SC_n \leq \alpha_n \leq PC_n + 2\lambda_A \psi \text{MCAR}_n;$$

band width = $PC_n + SC_n$

PC, SC = purchase and sale costs

Value added objective

$$\alpha_P - \lambda_A \psi_P^2 - \text{TC};$$

split aversions: $U = \alpha_P - (\lambda_{A,CF} \Psi_{P,CF}^2 + \lambda_{A,SP} \Psi_{P,SP}^2)$

CF = common factor, SP = specific

Dispersion bound

$$\psi^2 \leq \frac{\text{TC}}{2\lambda_A};$$

$$E\{r_{PA,\max} - r_{PA,\min}\} = 2 \times \Phi^{-1}\{(1/2)^{1/N}\} \times \psi$$

ψ and TC in percent

IM 5 Portfolio Risk: Analytical Methods

Diversified (portfolio) VaR

$$\text{VaR}_p = \alpha \sigma_p W = \alpha \sqrt{x' \Sigma x};$$

$$\text{two assets: } \text{VaR}_p = \alpha \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2} \times W$$

x = dollar exposures

Individual and undiversified VaR

$$\text{VaR}_i = \alpha \sigma_i |W_i| = \alpha \sigma_i |w_i| W;$$

$$\text{undiversified VaR} = \sum \text{VaR}_i;$$

$$\rho = 0: \text{VaR}_p = \sqrt{\text{VaR}_1^2 + \text{VaR}_2^2};$$

$$\rho = 1: \text{VaR}_p = \text{VaR}_1 + \text{VaR}_2$$

Equal-weight portfolio risk

$$\sigma_p = \sigma \sqrt{\frac{1}{N} + \left(1 - \frac{1}{N}\right) \rho} \rightarrow \sigma \sqrt{\rho} \text{ as } N \rightarrow \infty$$

Beta of position

$$\beta_i = \frac{\text{cov}(R_i, R_p)}{\sigma_p^2} = \rho_{ip} \frac{\sigma_i}{\sigma_p}; \quad \text{vector: } \beta = \frac{\Sigma w}{w' \Sigma w}$$

Marginal VaR

$$\Delta \text{VaR}_i = \frac{\partial \text{VaR}}{\partial x_i} = \alpha \frac{\text{cov}(R_i, R_p)}{\sigma_p} = \alpha \beta_i \sigma_p = \frac{\text{VaR}}{W} \beta_i$$

Incremental VaR

$$\text{Incremental VaR} = \text{VaR}_{p+a} - \text{VaR}_p \approx (\Delta \text{VaR})' \times a$$

a = vector of new positions

Component VaR

$$\text{CVaR}_i = (\Delta \text{VaR}_i) w_i W = \text{VaR} \beta_i w_i = \text{VaR}_i \rho_i;$$

$$\sum \text{CVaR}_i = \text{VaR}$$

Percent contribution to VaR

$$\frac{\text{CVaR}_i}{\text{VaR}} = w_i \beta_i$$

Best hedge (risk-minimizing trade)

$$a^* = -W \frac{\sigma_{ip}}{\sigma_i^2} = -W \beta_i \frac{\sigma_p^2}{\sigma_i^2}$$

Minimum-risk and optimal portfolio

$$\text{min risk: } \Delta \text{VaR}_i = \frac{\text{VaR}}{W} \times \beta_i = \text{constant (all } \beta_i \text{ equal);}$$

$$\text{max Sharpe: } \frac{E_i}{\Delta \text{VaR}_i} = \frac{E_i}{\beta_i} = \text{constant, } E_i = E_m \beta_i$$

E_i = excess return

IM 6 VaR and Risk Budgeting

Absolute vs relative VaR

$$\text{absolute: } \text{VaR} = \alpha W \sigma(R_{\text{asset}});$$

$$\text{relative: } E = R_{\text{asset}} - R^b, \quad \text{VaR} = \alpha W_0 \sigma_E$$

Policy mix vs active management return

$$R_{\text{asset}} = \sum w_i^b R_i^b + \sum (w_i R_i - w_i^b R_i^b)$$

first term policy mix, second active management

Surplus return (funding risk, SAR)

$$R_S = \frac{\Delta S}{A} = R_{\text{asset}} - R_{\text{liabilities}} \times \frac{L}{A}; \quad \text{SAR from } \alpha W \sigma_{\text{surplus}}$$

Risk budget per asset class

$$\text{risk budget}_i = \alpha \times \sigma_i \times (w_i W);$$

$$\sum \text{risk budgets} = \text{undiversified VaR} > \text{fund VaR}$$

Risk budgeting across active managers

$$\text{IR} = \frac{\mu}{\omega}; \quad \mu_p = \sum x_i \mu_i = \sum x_i (\text{IR}_i \times \omega_i);$$

$$\omega_p = \sqrt{\sum x_i^2 \omega_i^2}; \quad \text{optimal } x_i \omega_i = \text{IR}_i \times \frac{\omega_p}{\text{IR}_i};$$

$$\text{IR}_p = \sqrt{\sum \text{IR}_i^2}$$

ω = active risk; independent active returns

IM 7 Portfolio Performance Evaluation

Time-weighted vs dollar-weighted return

$$\text{TWR} = [(1 + r_1)(1 + r_2) \cdots (1 + r_n)]^{1/n} - 1;$$

$$\text{DWR} = \text{IRR: } \sum \frac{\text{CF}_t}{(1 + r)^t} = 0$$

TWR is the geometric average of period returns

Risk-adjusted measures

$$\text{Sharpe} = \frac{\bar{r}_P - \bar{r}_f}{\sigma_P}; \quad \text{Treynor} = \frac{\bar{r}_P - \bar{r}_f}{\beta_P};$$

$$\text{Jensen } \alpha_P = \bar{r}_P - [\bar{r}_f + \beta_P(\bar{r}_M - \bar{r}_f)]; \quad \text{IR} = \frac{\alpha_P}{\sigma(e_P)}$$

M²

$$M_P^2 = r_{P^*} - r_M$$

P^* = mix of P and T-bills with $\sigma_{P^*} = \sigma_M$ (weight in P = σ_M/σ_P)

T² (Treynor square)

$$T^2 = r_{P^*} - r_M$$

P^* = mix of P and T-bills with $\beta_{P^*} = 1$ (weight in P = $1/\beta_P$)

Sharpe gain from active portfolio

$$S_P^2 = S_M^2 + \left[\frac{\alpha_H}{\sigma(e_H)} \right]^2$$

Standard error and t-statistic of alpha

$$\hat{\sigma}(\alpha) = \frac{\hat{\sigma}(e)}{\sqrt{N}}; \quad t(\hat{\alpha}) = \frac{\hat{\alpha}}{\hat{\sigma}(\alpha)} = \frac{\hat{\alpha}\sqrt{N}}{\hat{\sigma}(e)}$$

Market timing regressions

$$\text{Treynor-Mazuy: } r_P - r_f = a + b(r_M - r_f) + c(r_M - r_f)^2 + e_P;$$

$$\text{Henriksson-Merton: } r_P - r_f = a + b(r_M - r_f) + c(r_M - r_f)D + e_P;$$

$$D = 1 \text{ if } r_M > r_f$$

$c > 0$ means timing ability

Value of market timing (call option)

$$\text{perfect timer: } C = 2N\left(\frac{1}{2}\sigma_M\sqrt{T}\right) - 1 \text{ per dollar of assets;}$$

$$\text{imperfect: } MV = (P_1 + P_2 - 1) \times C$$

$$\text{timing ability} = P_1 + P_2 - 1$$

Performance attribution

$$\text{bogey} = \sum w_{Bi}r_{Bi}; \quad \text{excess} = r_P - r_B;$$

$$\text{asset allocation} = \sum (w_{Pi} - w_{Bi})r_{Bi};$$

$$\text{selection} = \sum w_{Pi}(r_{Pi} - r_{Bi});$$

$$\text{sector allocation within equity} = \sum (\text{active sector weight} \times \text{sector return})$$

IM 8 Hedge Fund Investment Strategies

Merger arbitrage annualized return

$$\text{Exp. annualized return} = \frac{C \times G - L \times (1 - C)}{Y \times P}$$

C = probability of completion; Y = years to close; P = current price;

G = offer - current price; L = current price - fallback price

Convertible arbitrage income

$$\text{income} = \text{coupon} + \text{interest on short proceeds} - \text{stock dividend} - \text{stock borrow cost};$$

$$\text{short proceeds} = \text{share price} \times \text{conversion ratio} \times \% \text{ shorted}$$

shares shorted depend on delta $\times$ conversion ratio

IM 11 Private Markets Investing

Multiples

$$\text{DPI} = \frac{\sum \text{distributions}}{\text{paid-in}}; \quad \text{RVPI} = \frac{\text{residual value}}{\text{paid-in}};$$

$$\text{TVPI} = \text{DPI} + \text{RVPI} = \frac{\sum \text{distributions} + \text{residual value}}{\text{paid-in}}$$

$$\text{maturity} = \text{DPI} / \text{TVPI}$$

Public market equivalent

$$\text{PME} = \frac{\sum \text{discounted distributions}}{\sum \text{discounted capital calls}}$$

discounted with the index total return;
PME > 1 means outperformance

IM 15 Market-Driven Scenarios

Mahalanobis distance and scenario z-score

$$\text{MD}(r, \Sigma) = \sqrt{(r - \bar{r})' \Sigma^{-1} (r - \bar{r})}; \quad Z(r, \Sigma) = \frac{\text{MD}(r, \Sigma)}{\sqrt{n}}$$

n = number of policy variables; lower Z = more plausible

Volatility and correlation z-scores

$$V(z) = \sqrt{\frac{z'z}{n}}; \quad C(r, \Sigma) = \frac{Z(r, \Sigma)}{V(z)} = \sqrt{\frac{z'}{\sqrt{z'z}} \Lambda^{-1} \frac{z}{\sqrt{z'z}}}$$

z = vector of factor z-scores (shock/ σ); Λ = correlation matrix;
 $C > 1$: shocks inconsistent with the correlations

IM 16 Liquidity Risk Management

Days to liquidate

$$\text{Days to liquidate} = \frac{\text{size of trade}}{\text{participation rate} \times \text{ADV}}$$

Expected volume, infrequently traded bond

$$E(V_{t+1}^b) = p_{t+1}^b \times E(V_{t+1}^b | \text{TRADE})$$

p^b = probability the bond trades

T-cost model

$$\text{t-cost} = [k_1 \times \text{BAS}] +$$

$$\left[k_2 \times D \times S \times \left(\frac{\text{trade notional volume}}{\text{ADV}} \right)^\gamma \right]$$

fixed cost plus market impact; BAS = % bid-ask spread;
 D = spread duration; S = OAS

Implementation shortfall

$$\text{IS (bp)} = \text{sign} \times \frac{\text{traded price} - \text{benchmark price}}{\text{benchmark price}} \times 10,000$$

sign = +1 buy, -1 sell

IM 17 Illiquid Assets

Unsmoothing (Geltner-Ross-Zisler)

$$r_t = \frac{1}{1 - \varphi} r_t^* - \frac{\varphi}{1 - \varphi} r_{t-1}^*;$$

$$\text{equivalently } r_t^* = (1 - \varphi) r_t + \varphi r_{t-1}^*$$

r^* = reported, r = true, φ = autocorrelation of reported returns

Effect on volatility

$$\text{var}(r_t) \geq \text{var}(r_t^*)$$

unsmoothed volatility is higher; means unchanged